Open Problems from CCCG 2000

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The following is a list of the problems presented on August 18, 2000 at the open-problem session of the 12th Canadian Conference on Computational Geometry. One problem posed at the session was later withdrawn when it became clear subsequently that it was already solved in the literature. One other problem, posed during N. Katoh's invited talk (and not reflected in his printed abstract), is also included.

Recognizing shortest path trees
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This problem is motivated by J. O'Rourke's talk at CCCG 2000 [KO00] in which he showed many pictures of shortest path trees on three-dimensional polyhedra, and told stories how when testing the implementation of their algorithm, they often simply looked at pictures and "saw" whether the tree could be correct. This lead me to wonder about the following question:

> Given a tree with coordinates for the nodes, how easy is it to determine whether this tree could be a shortest-path tree for a simple polyhedron?

Several variations could be studied here:

- In what dimension is the tree? 2D? 3D? Even higher?
- Do we know the root of the tree?
- If the tree is a shortest-path tree, is it easy to reconstruct the polygon/polyhedron? Is the polygon/polyhedron unique?

Update: T. Biedl and J. O'Rourke studied this problem during CCCG 2000, and made some progress for the two-dimensonal case. The three-dimensional case remains open. Results for 2D (see Fig. 1) are as follows:

- If the root is given, and we also know the two incident edges of the polygon that attach to the root, then we can test in $O(n)$ time whether the tree is the shortest-path tree of a simple polygon. Moreover, this polygon is unique and can be found in the process.

  (This result was discovered independently by Ileana Streinu, using techniques developed for characterizing visibility graphs in [OS97].)
- If the root is given, but the two incident edges of the polygon are not given, then there may be as many as n different simple polygons for which the tree is a shortest-path tree. However, this can only happen if the incident edges of the root are not contained within a half-space, i.e., the root is at a reflex vertex of the polygon.

  We can still test in $O(n^2)$ time whether such a simple polygon exist. We suspect that $O(n)$ time is possible.
- If the root is not given, then there may be as many as $\Omega(n)$ different vertices of the tree that are the root in some shortest path tree of a simple polygon.

  However, this case and the previous case cannot happen simultaneously, which means that we still have an $O(n^2)$ algorithm to find a simple polygon, if one exists. Again we suspect that this can be improved to $O(n)$.

[KO00] Biliana Kaneva and Joseph O'Rourke, "An implementation of Chen & Han's shortest paths algorithm," *Proceedings of the 12th Canadian Conference on Computational Geometry*, 2000, pages 139–146.

[OS97] Joseph O'Rourke and Ileana Streinu, "The vertex-edge visibility graph of a polygon," *Comput. Geom. Theory Appl.*, 8: 1–17 (1997).

Self-Adjusting Planar Point Location
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This problem asks for a planar point-location data structure that achieves entropy bounds analogous to splay trees for one-dimensional queries. More precisely, we are given n triangular regions in the plane, $r_1, \ldots, r_n$. Although unknown to the algorithm, there is an underlying distribution on the

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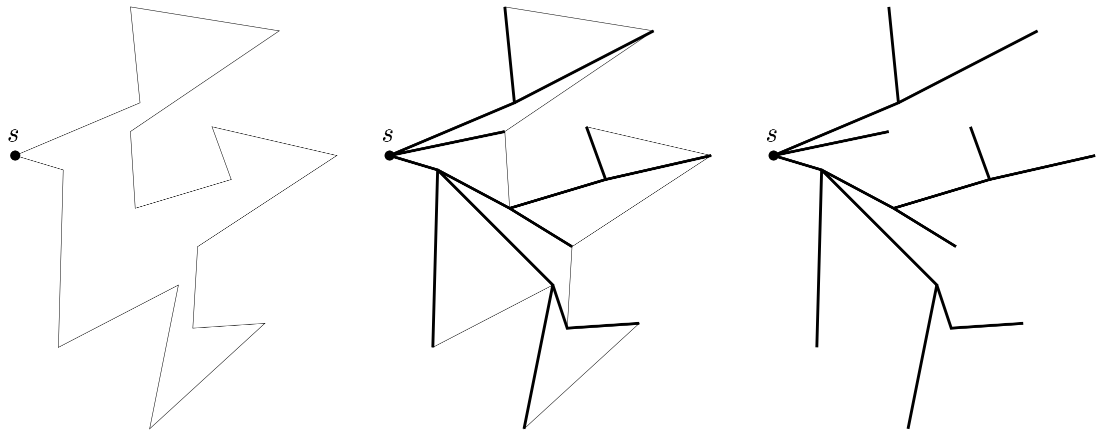

Figure 1: A polygon with source s; together with the shortest path tree; given the shortest path tree alone, can the polygon be reconstructed?

queries: m_i queries will be made to region r_i. Let M be the total number of queries, $\sum_{i=1}^{n} m_i$. Is there a data structure supporting the queries in $O(n \log n + \sum_{i=1}^{n} m_i \log(M/m_i))$ time?

It is known that this bound is achievable if the query distribution is available to the algorithm, and that the bound is unachievable if the regions are convex polygons instead of triangles.

Point Location in Linearly Moving Regions
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Given a planar graph structure defined on n points, is it possible to efficiently maintain a planar point-location structure as the points move along linear trajectories? In particular, consider the Kirkpatrick structure for planar point-location in triangulations. Given a triangulation of n points in the plane, is it possible to maintain an independent set of $\Omega(n)$ points as the points move on linear trajectories?

Asymmetric Farthest-Point Voronoi Diagram
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This problem arises in the context of circular ray shooting [BU00].

The *asymmetric farthest-point Voronoi diagram* of a set of points is the farthest-point Voronoi diagram according to the following measure: the distance between point a and point b is the Euclidean distance if a is at or below b in y coordinate, and zero otherwise. Because this distance function is not a metric, the resulting Voronoi diagram can be complex; in particular, it may have $\Omega(n^2)$ edges [BU00]. Conjecture: For random uniformly distributed points, the size of the asymmetric farthest-point Voronoi diagram is $O(n \log n)$. See [BU00] for further details.

Update: Hamish Carr and Ralph Boland made progress on this problem at CCCG 2000, but the details are not yet clear.

[BU00] Ralph P. Boland and Jorge Urrutia, "Circular ray shooting: Slower but surer," *Proceedings of the 12th Canadian Conference on Computational Geometry*, 2000, pages 37–42.

Balanced Determined Lines
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The following problems come from Y. Kupitz.

Consider a finite set of points in the plane not all on a common line. A line is *determined* if it contains at least two of the points.

(1) What is the smallest r such that, for any set of $2n+r$ points, there is a determined line having at least n points in each open halfplane? In particular, is $r = O(1)$? The best current bound is $O(\log \log n)$.

(2) What is the smallest t for which there is always a determined line such that the difference between the number of points on its two sides is at most t?

Periodic Billiard Paths
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Does every triangle have a periodic billiard path? A billiard ball is a point that travels along a straight line inside the polygonal billiard table, bouncing off the sides with perfect elastic reflection, just as a light ray from a mirror. A period billiard path is one that repeats, rejoining its earlier

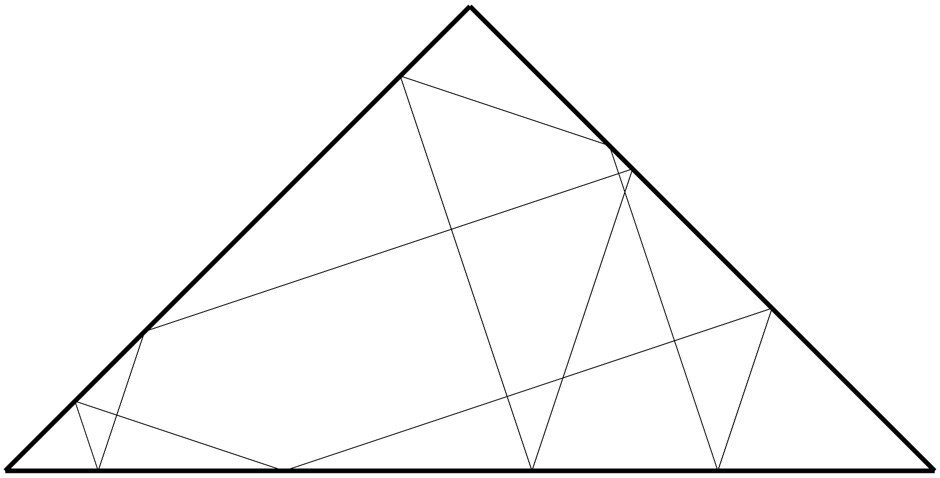

Figure 2: A periodic billiard path in a triangle.

path; see Fig. 2. This is an old problem, perhaps going back to Birkoff in 1927; it is Problem 1.4 in [KW91]. It is known that every right triangle has such a path; in fact, almost every trajectory that starts orthogonal to a side of the right angle is periodic. And every acute triangle has a periodic trajectory represented by its "Fagnano triangle" (also known as the "orthic triangle"), connecting the feet of the three altitudes. A few other special classes are known to possess periodic paths, but the question for general, obtuse triangles is unsolved. See [Tab95], and [HH00] for the most recent results.

[**HH00**] Lorenz Halbelsen and Norbert Hungerbühler, "On periodic billiard trajectories in obtuse triangles," *SIAM Review*, 42(4):657–670, 2000.

[**KW91**] Victor Klee and Stan Wagon, *Old and New Unsolved Problems in Plane Geometry*, Mathematical Association of America, 1991.

[**Tab95**] Serge Tabchnikov, *Billiards.* Société Mathématique de France. Panoramas et Synthéses 1. 1995.

Minimum Corridor Connection

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Given a rectilinear decomposition of a rectilinear polygon (a subdivision into *rooms*), find the minimum-length tree along the edges of the decomposition such that every room is incident to a vertex of the tree. See Fig. 3. Is this problem NP-complete? Are there efficient approximation algorithms?

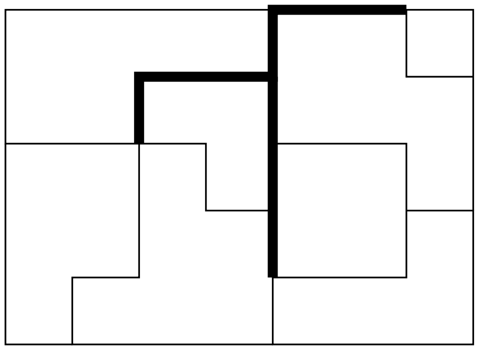

Figure 3: Minimum-length tree among the corridors to connect the rooms.