Open Problems from CCCG 2002

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The following is a list of the problems presented on August 12, 2002 at the open-problem session of the 14th Canadian Conference on Computational Geometry held in Lethbridge, Alberta, Canada.

Boxed problem numbers indicate appearance in The Open Problem Project (TOPP); see `http://www.cs.smith.edu/~orourke/TOPP/`.

Great Circle Graphs: 3-colorable?

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TOPP #44

Is every zonohedron 3-colorable when viewed as a planar map? This question arose out of work described in [RSW01]. An equivalent question, under a different guise, is posed in [FHNS00]: Is the arrangement graph of great circles on the sphere 3-colorable? Assume no three circles meet at a point, so that this graph is 4-regular. Circle graphs in the plane can require four colors [Koe90], so the key property in this problem is that the circles must be great. All arrangement graphs of up to 11 great circles have been verified to be 3-colorable by Oswin Aichholzer (August, 2002). See [Wag02] for more details.

References

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Kissing Circle Representation

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It is known that any planar graph G can be represented by "kissing circles": an interior-disjoint collection of circles, one circle per vertex, such that two circles touch ("kiss") precisely when the corresponding vertices are adjacent. However, the computation of such kissing circles is not straightforward. See [Koe35, Moh93, BS93, Sac94, Smi91, Zie95] for more information.

Suppose one loosens the kissing requirement and seeks instead a collection of disks whose intersection graph is G. Is it easier to compute such a representation? Can the disk centers be restricted to rational coordinates? Can they be integers bounded by a polynomial in some parameters of the graph?

References

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3-Manifolds Built of Boxes
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A result in [DO01, DO02] may be interpreted as follows: For any polyhedral 2-manifold homeomorphic to a sphere $\mathbb{S}^2 \subset \mathbb{R}^3$, all of whose facets are rectangles, adjacent facets either meet orthogonally or are coplanar. This raises the analogous question one dimension higher: For any polyhedral 3-manifold homeomorphic to a sphere $\mathbb{S}^3 \subset \mathbb{R}^4$, all of whose facets are rectangular boxes, is it true that adjacent facets lie either in orthogonal 3-flats or within the same 3-flat? Very roughly, must a 3-manifold built from boxes be itself orthogonal?

References

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Visibility Product Characterization
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Let P be a polygon, treated as a region in the plane. Define (for lack of a better term) the *visibility product* $\mathrm{VP}(P)$ to be the following four-dimensional set:

$$\begin{aligned}\mathrm{VP}(P) = \{(x_1, y_1, x_2, y_2) \mid \\ (x_1, y_1) \in P,\ (x_2, y_2) \in P, \\ (x_1, y_1) \text{ can see } (x_2, y_2)\}\end{aligned}$$

Two points can *see* one another if the line segment between those points is a subset of P. Thus VP is something like a set product capturing visibility. Determine the structure of $\mathrm{VP}(P)$, characterize the set, find an algorithm to construct it, and determine if it has utility.

3D Orthogonal Graph Drawings
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TOPP #46

Does every simple graph with maximum vertex degree $\Delta \leq 6$ have a 3D orthogonal point-drawing with no more than two bends per edge? An *orthogonal point-drawing* of a graph maps each vertex to a unique point of the 3D cubic lattice, and maps each edge to a lattice path between the endpoints; these paths can only intersect at common endpoints. In this problem, each path must have at most two bends, that is, consist of at most three orthogonal line segments (links).

There are several related known results. Two bends would be best possible, because any drawing of K_5 uses at least two bends on at least one edge. If $\Delta \leq 5$, two bends per edge suffice [Woo03]. Two bends also suffice for the complete multipartite 6-regular graphs K_7, $K_{2,2,2,2}$, $K_{3,3,3}$, and $K_{6,6}$ [Woo00]. In general, there is a drawing with an average number of bends per edge of at most $2 + \frac{2}{7}$ [Woo03]. Additionally, three bends per edge always suffice, even for multigraphs [ESW00, PT99, Woo01].

This problem was first posed in [ESW00].

References

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Sailor-in-the-Fog Generalization
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The venerable "Sailor in the Fog" problem asks for an optimal search strategy for a sailor to find the shoreline when lost in a fog offshore (a version was posed by Bellman in [Bel87]). There are many variations on this problem. For example, one version can be rephrased as follows: Find the shortest-length path from the center of a unit disk that intersects every halfplane whose bounding line (the shoreline) supports the disk. Note here the assumption is that the distance to the shore is known. This problem was solved by Isbell [Isb57].

A conference conversation suggested the following higher-dimensional generalization: Find the shortest-length path from the center of a unit ball that intersects every halfspace whose bounding plane supports the ball. This problem might represent a diver seeking the surface.

It came to light after the presentation that this problem was posed before, in a paper by V. A. Zalgaller [Zal92], for which there is apparently no published translation from the Russian. Nonetheless, the problem remains unsolved. See [Fin01] for more information.

References

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Region Realization

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Suppose we are given a collection of constraints on unknown planar connected regions of the form

1. region A is contained in region B; and
2. region A properly intersects region B;
3. regions A and B are adjacent (share just boundary points);
4. region A does not touch region B.

Is there a polynomial-time algorithm to decide whether there is a realization of these constraints by planar connected regions? The special case involving just constraints of Type 3 is called a *map graph*, a concept introduced by Chen, Grigni, and Papadimitriou [CGP02] and solved by Mikkel Thorup [Tho98].

A simpler variation on this problem is that all regions are given (as planar polygons) except for one unknown region X which must be found in order to obey the given constraints.

References

[CGP02] Zhi-Zhong Chen, Michelangelo Grigni, and Christos H. Papadimitriou. Map graphs. *Journal of the ACM*, 49(2):127–138, 2002.

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Guaranteed Aspect Ratio Partitions

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Define the *aspect ratio* of a polygon as the ratio of the diameters of the smallest circumscribing circle to the largest inscribed circle. (Thus in this context the aspect ratio measures circularity.) Find a polynomial-time algorithm for partitioning a polygon into the fewest polygonal pieces, each piece with an aspect ratio no more than a given $\alpha > 1$, or to report that no such partition exists. Here the pieces are permitted to employ "Steiner points," points that are not vertices of the given polygon. When Steiner points are disallowed, a polynomial-time algorithm is known [DI02]. A second question is to find the smallest $\alpha > 1$ for which there is a partition in which every piece has aspect ratio at most α.

References

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Representing Separation by Pseudotriangulation

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Consider a nonoverlapping collection of polygons in the plane. Is it always possible to decompose the exterior of these polygons into pseudotriangles such that each object touches at most as many pseudotriangles as its minimum-link separation chain? How efficiently can such a decomposition be computed, when it exists?

D-forms

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Let c_1 and c_2 be two smooth, closed, convex, planar curves of the same length, each bounding a flat piece of paper. Choose a point p_1 on c_1 and a point p_2 on c_2, and glue the two curves to each other (according to arclength) starting with p_1 glued to p_2. The resulting single piece of paper forms a shape in space called a *D-form* by Helmut Pottmann, Johannes Wallner, and Tony Wills. The curves c_1 and c_2 join to form a closed space curve $\mathbf{c}$ bounding two developable surfaces S_1 and S_2. These authors ask two questions in [PW01, p. 418]:

1. "It is not clear under what conditions a D-form is the convex hull of a space curve."
2. "After some experiments we found that, surprisingly, both S_1 and S_2 were free of creases, but we do not know whether this will be so in all cases."

References

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