Linkage Folding: From Erdős to Proteins

Erik D. Demaine
Computer Science and Artificial Intelligence Laboratory
Massachusetts Institute of Technology
Cambridge, Massachusetts

Abstract

Linkages have a long history ranging back to the 18th century in the quest for mechanical conversion between circular motion and linear motion, as needed in a steam engine. In 1877, Kempe wrote an entire book of such mechanisms for "drawing a straight line". (In mathematical circles, Kempe is famous for an attempted proof of the Four-Color Theorem, whose main ideas persist in the current, correct proofs.) Kempe designed many linkages which, after solidification by modern mathematicians Kapovich, Millson, and Thurston, establish an impressively strong result: there is a linkage that signs your name by simply turning a crank.

Over the years mathematicians, and more recently computer scientists, have revealed a deep mathematical and computational structure in linkages, and how they can fold from one configuration to another. In 1936, Erdős posed one of the first such problems (now solved): does repeatedly flipping a pocket of the convex hull convexify a polygon after a finite number of flips? This problem by itself has an intriguing history, culminating at this CCCG. A surge of results over the past few years have intriguing applications to robotics, graphics, nanomanufacture, and protein folding.