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HIDING DISKS IN FOLDED POLYGONS

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Joint work with

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http://daisy.uwaterloo.ca/~eddemain/papers/CCCG98a/

INTRODUCTION

- Gift wrapping problem:

  Can polyhedron Q be wrapped by folding polygon P around it?

- One dimension down:

  Can 2-D shape Q be hidden/doubly covered by folding polygon P around it?

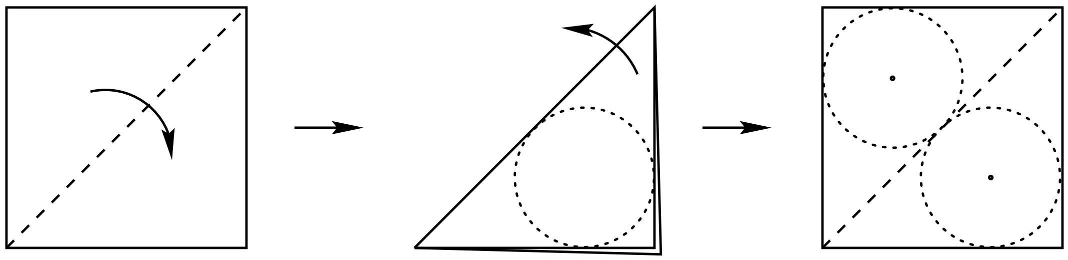
- Optimization version:

  What is the largest scaling of shape Q that can be hidden by folding P around it?

- Here we solve the case of
  - Q a disk
  - One simple fold = fold along a line

PACKING FORMULATION

- Consider hiding a disk with one simple fold:



  - In the original polygon, this gives two nonoverlapping copies of a disk

- The other direction:
  - Take any two nonoverlapping copies of a disk
  - Fold along the perpendicular bisector of their centers
  - Obtain a disk hiding

- Hence the disk-hiding problem is equivalent to

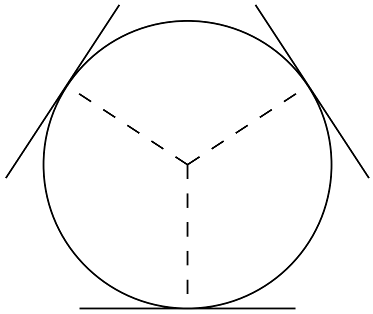
  Find the largest pair of equal-radius nonoverlapping disks in a polygon P.

RELATED PACKING PROBLEMS

- General problem:

  Find the largest k equal-radius nonoverlapping disks in a polygon P.

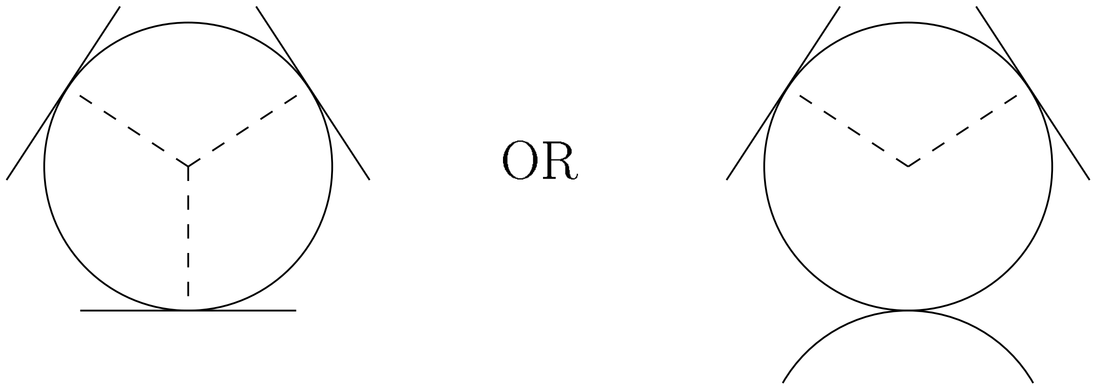
- Solved for small k in special polygons
- Solved for $k = 1$ in general polygons (Shamos & Hoey 1975)
  - Center of the disk is on a vertex of the medial axis



- Here show a solution for $k = 2$ in general polygons

RELATION TO MEDIAL AXIS

- Consider maximal pair of equal-radius nonoverlapping disks
  - One disk touches ≥ 3 objects (disk or polygon edge)



- At most one object can be a disk
- Hence, center is on the medial axis
- Can also move the other disk to put its center on the medial axis
- Further, if the disks don't touch, one is the largest disk centered at a vertex of the medial axis

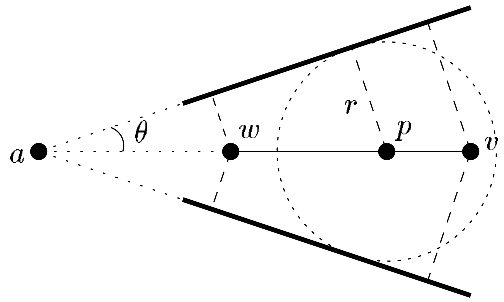
BASIC ALGORITHM

1. Compute medial axis of input polygon
2. For each vertex v and edge e on the medial axis:
   - Find largest disk D centered at v
   - Check if a copy of D can be centered on e while not overlapping D and staying inside P
3. For each edge e_1 and edge e_2 on the medial axis:
   - Find centers c_1 on e_1 and c_2 on e_2 to maximize the radius
     $$r = \frac{|c_1 - c_2|}{2}$$
     such that the (touching) disks are inside P.
4. Return the largest pair of disks found

PARAMETERIZING BY RADIUS

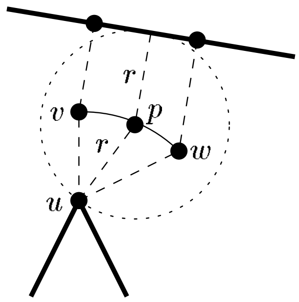
- Parameterize a point on an edge of the medial axis by radius of largest disk in P centered at that point
  - Line-segment edges:

$$q + r\ \vec{v}$$



  - Parabolic edges:

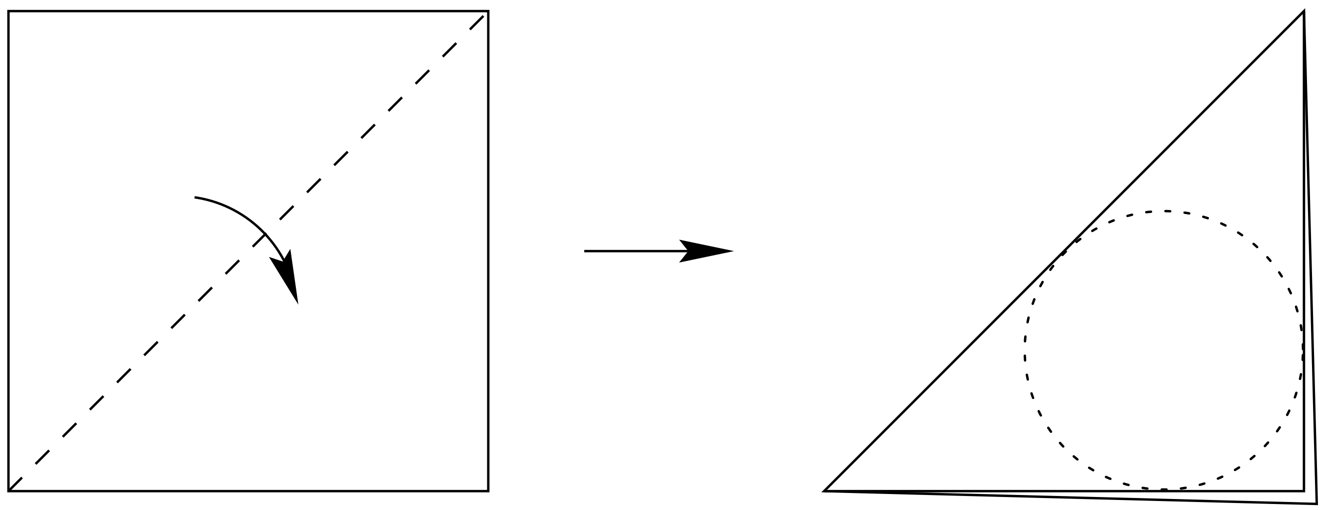
$$q + r\ \vec{v}_1 + \sqrt{r - c}\ \ \vec{v}_2$$



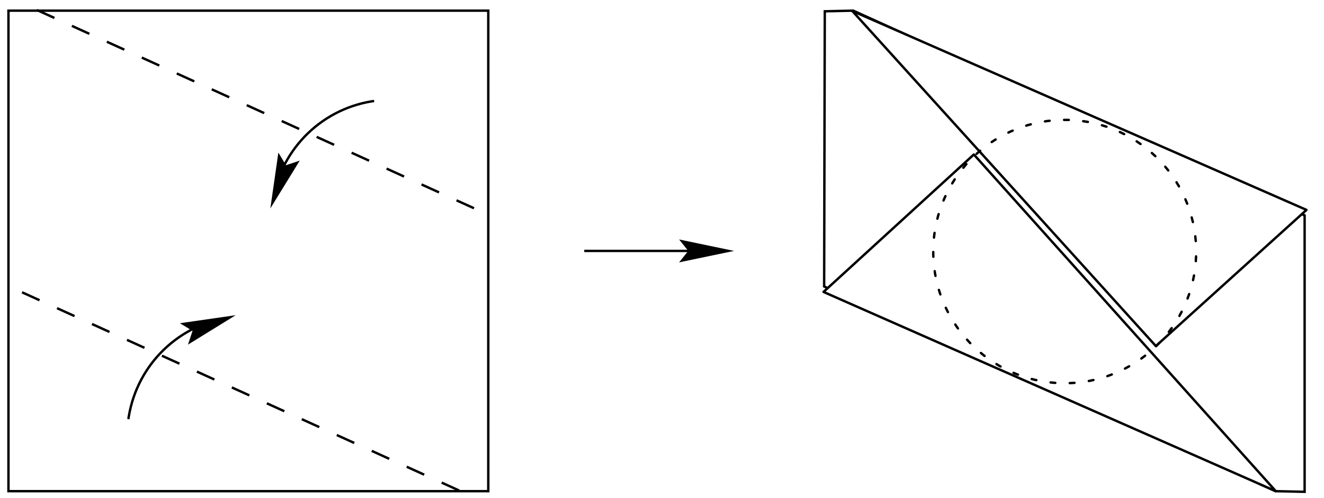
- Denote point on edge e with corresponding radius r by $e(r)$
- Nontouching case is easy
- Touching equation: $|e_1(r) - e_2(r)| = 2r$
  - For convex polygons, degree 2 polynomial in r
  - In general, degree 8 polynomial in r

SQUARE CASE

One fold: $r = \frac{1}{\sqrt{2}} \tan \frac{\pi}{8} \approx 0.29289$



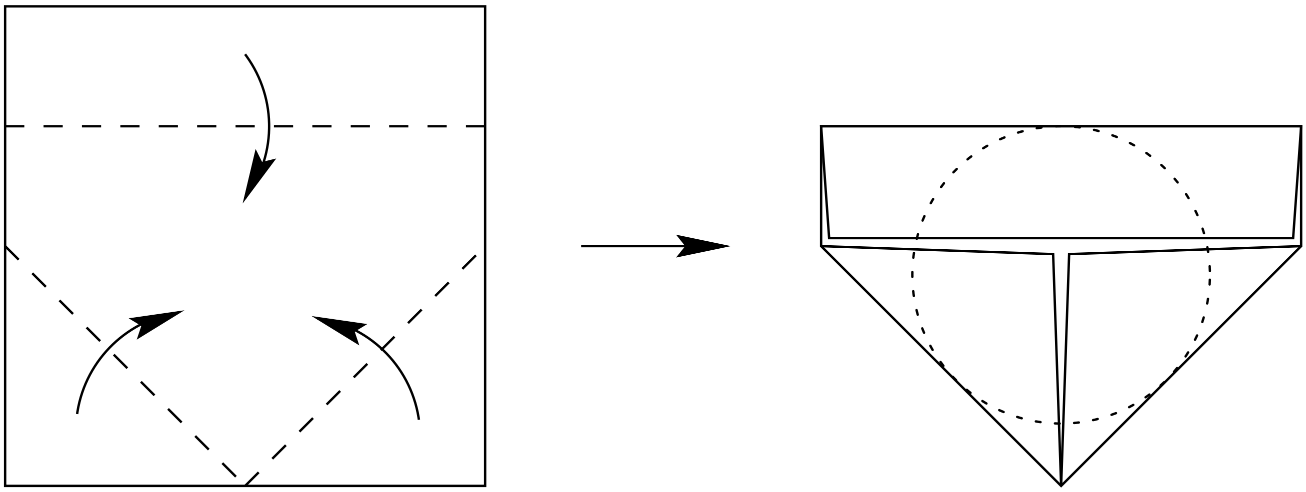
Two folds: $r = \frac{\sqrt{7}-1}{6} \approx 0.27429$



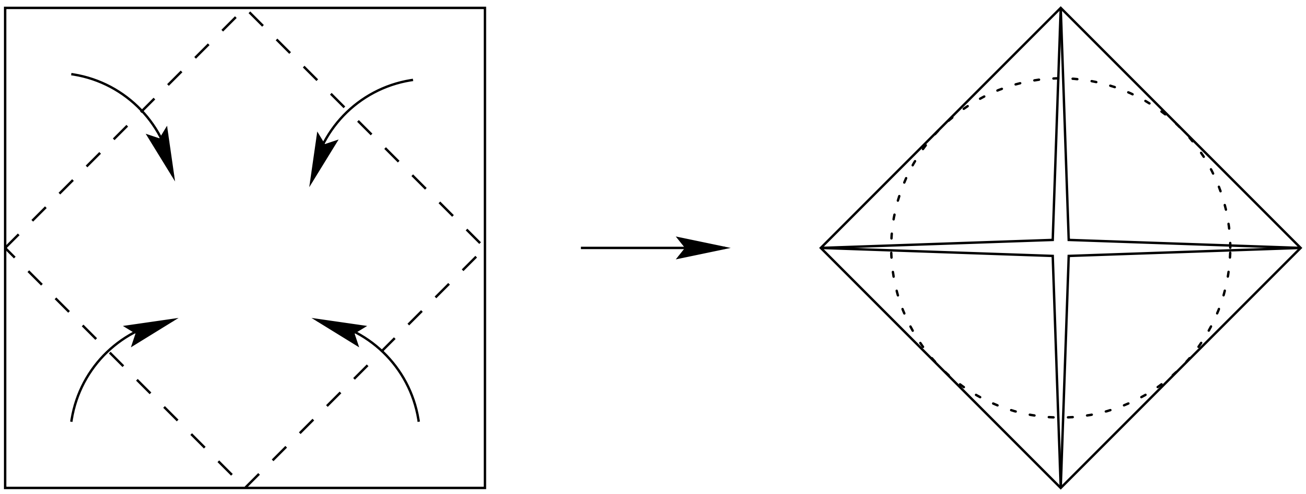
SQUARE CASE

# folds	1	2	3	4
Radius	0.29289	0.27429	0.31066	0.35355

Three folds: $r = \frac{3}{4}\tan\frac{\pi}{8} \approx 0.31066$



Four folds: $r = \frac{\sqrt{2}}{4} \approx 0.35355$



CONCLUSION

- Polynomial-time algorithm for hiding disks using one simple fold
- Conjectures for multiple folds of a square

OPEN PROBLEMS

- Multiple folds
  - Can foldings for the square be improved?
- Arbitrary foldings
- Hiding circles
- Find largest scaling of polygon Q that can be hidden by P
- Higher dimensions