

# Curved Crease Origami

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## Abstract

Most origami, both practical and mathematical, uses just straight creases. Curved creases, on the other hand, offer a wealth of new design possibilities. While the first curved-crease models date back to the Bauhaus in the 1930s, curved creasing remains relatively underexplored. The principal challenge considered here is to understand what 3D forms result as natural resting state(s) after folding a set of curved creases, with the potential to enable a new category of design. This problem goes beyond the mathematics of developable surfaces to a question of physics: equilibria of an unstretchable surface with uncreased and creased (plastically deformed) portions folding elastically toward desired angles. Two natural approaches for experimenting with this question are computer simulation and building real models. We follow the latter approach, being more interested in how real materials behave and how the resulting structures might be applied in the field of architecture.

**Keywords:** architecture, mathematical origami, curved creases, developable surfaces

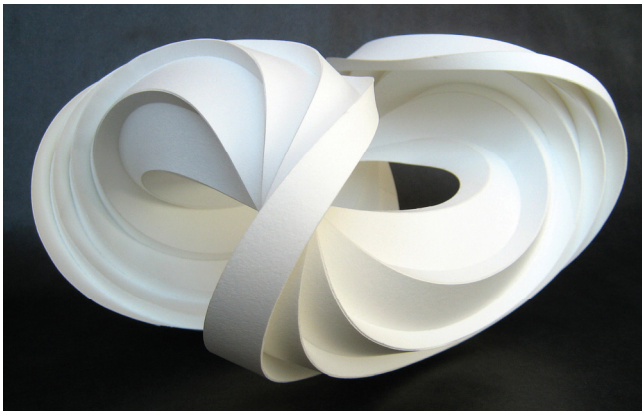


Figure 1: circles & ellipses; offset

## 1 Introduction

Most materials used for dry building enclosures are supplied as sheet goods, making developable surfaces—surfaces foldable from a flat sheet—the geometry of choice [She02]. Nondevelopable curved surfaces are made primarily by casting, stamping, or similar methods that need a dye or mold, which lacks economy of scale if the individual components are different from each other.

This research proposes a family of curved three-dimensional geometries that can be fabricated from two-dimensional sheet materials, by way of curved creases; see Figure 1. We also

show proofs of concept for fabricating such shapes in materials suitable for architectural applications.

## 2 Academic Context

The first known reference of curved-crease origami is from a student’s work at the Bauhaus, taking a preliminary course in paper study by Josef Albers in 1927–1928 [Win69, p. 434]. Albers later taught the model—formed from creasing a circular piece of paper with concentric circles, alternating mountain and valley—at Black Mountain College circa 1937–1938 [Adl04, p. 33, p. 73]. Irene Schawinsky (wife of Alexander “Xanti” Schawinsky) developed a variation with a central concentric circular hole, exhibited at the Museum of Modern Art (MoMA) in New York [McP44, p. 42]. Later this model entered origami circles through Thoki Yenn from Denmark and Kunihiko Kasahara from Japan. More intricate curved-crease origami sculpture has been designed by Ronald Resch (1970s), David Huffman (1970s–1990s), Jeannine Mosely (2000s), Gregory Epps (2000s), and Demaine and Demaine (2000s); see [DD] for a recent MoMA exhibition and a more detailed history.

The mathematical literature encompasses a reasonable understanding of how curved creases can fold locally; see, for example, Huffman’s one paper [Huf76] and the more recent works [FT99, KFC+08]. However, there is essentially no algorithmic understanding of how to design origami using curved creases, unlike the wealth of algorithms for straight creases; see [DO07]. We aim to start filling this gap by experimenting with a range of designs.

Part of the challenge is that the three-dimensional forms taken by curved-crease origami are not usually determined mathematically: treated mechanically, the models have many degrees of freedom. Yet physical paper prefers to rest in one (or a few) stable equilibria. These equilibria (locally) minimize the elastic energy of the system: where paper is uncreased, it tries to return to its original flat state; and where paper has been creased (plastically deformed, effectively modifying its memory), it tries to return to the set crease angle. (Exactly how far the crease-angle memory is set depends on how hard one folds the creases, which affects the final form.) Physics balances these forces, often resulting in surprising three-dimensional forms.

Being difficult to solve analytically, we can find this family of natural folded forms by either physical experiment or computer simulation. Computer simulation of origami [KKG94, MYYT96, BGW06, Tac07, KWC] has so far focused on straight creases, in some cases allowing developable surfaces between straight creases [MYYT96, BGW06] and in one case allowing curved creases [KKG94]; others have tested using piecewise-straight approximations of curved creases [Tac07]. Only a few, however, simulate actual physics of paper [BGW06, KWC]. We opt for an experimental approach both to ground any future computer simulation and to better understand any influence of the material choice (not modeled by these simulators).

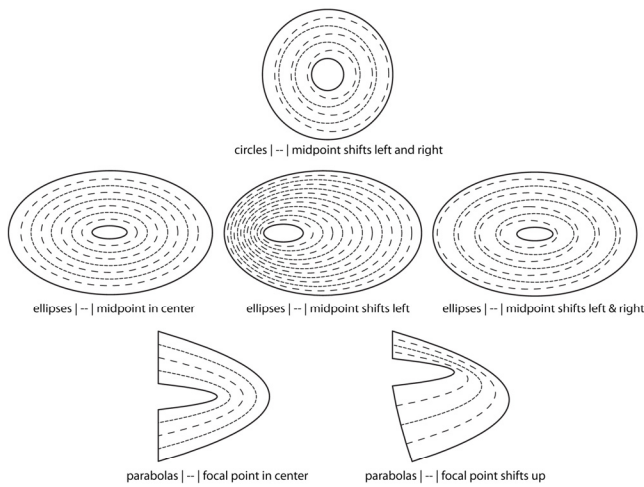


Figure 2a: circles, ellipses, parabolas

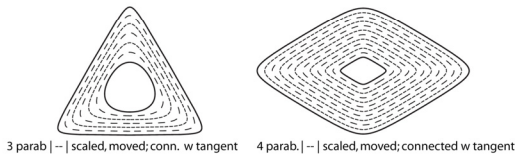


Figure 2b: combined parabolas

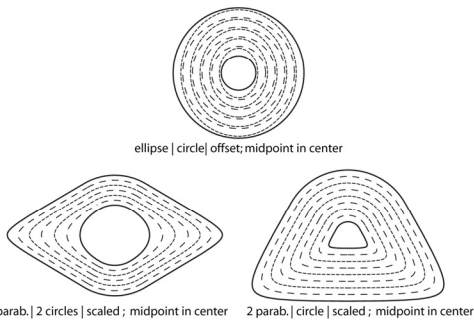


Figure 2c: combined circles, ellipses & parabolas

### 3 Experiments

We consider curved crease patterns consisting of several regular offsets of a variety of different piecewise-quadratic smooth curves, with fold directions alternating between mountain and valley. In an origami context, such crease patterns correspond to “pleating”, and they naturally extend the Bauhaus form of concentric circles. Specifically, we consider circles, ellipses, and parabolas, both whole and joined together in pieces, mostly to form closed loops. The offsets we consider are concentric, shifting monotonically in one direction, and shifting alternately back and forth in one direction.

Figures 2a–2c show some of the drawn patterns of our experiments. A total of 20 shapes were tested successfully. Only 11 are documented here because of similarities in crease patterns and resulting three-dimensional form. Our experiments use a cotton-based paper, scored on each side with a laser cutter.

Several interesting and sometimes unexpected phenomena

arose from our experiments. Perhaps most exciting is the wide variety of three-dimensional forms resulting from sometimes subtly different crease patterns, leaving a broad spectrum for design even within the context of pleating. Also intriguing is that shifting offset ellipses (as well as circles) alternately back and forth along a line, as shown in Figure 3 and 4c, results in a “twisted” folded form that lacks the mirror symmetry of the crease pattern. In contrast, shifting offset ellipses monotonically in one direction results in a mirror-symmetric form, as shown in Figure 4b.

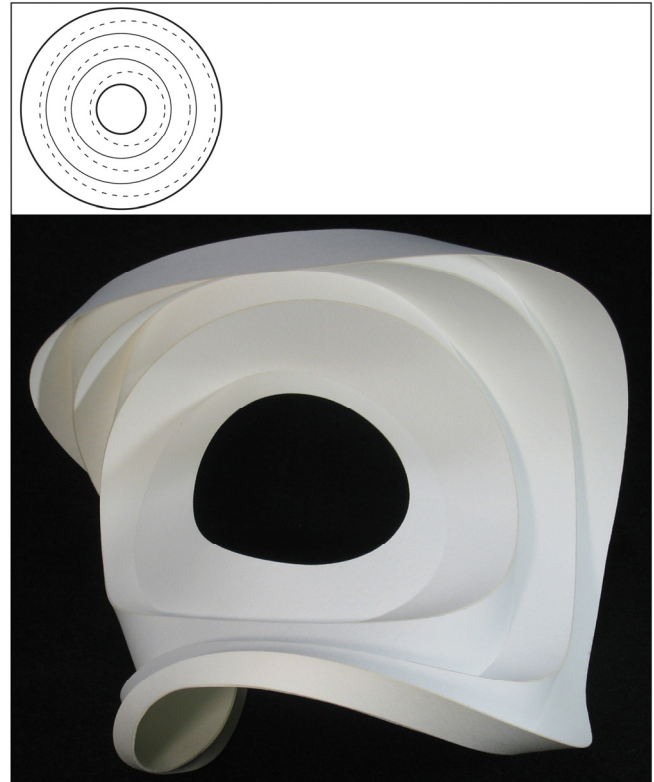


Figure 3: circles; center shifts left and right

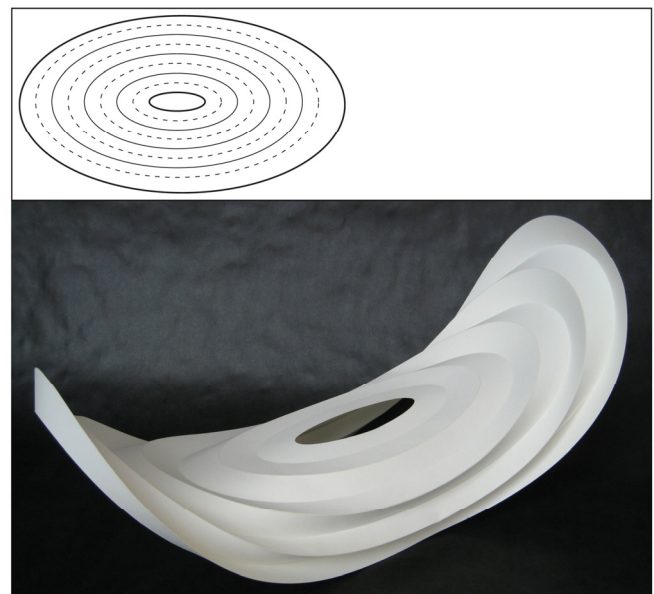


Figure 4a: ellipses; offset on center



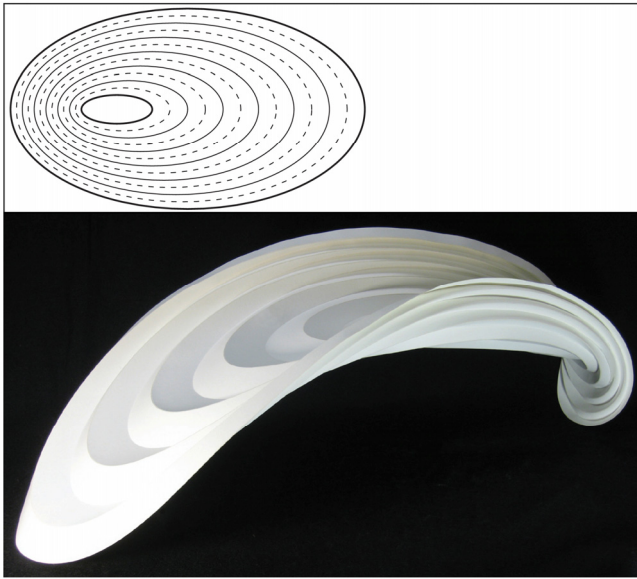


Figure 4b: ellipses; offset, center shifts left

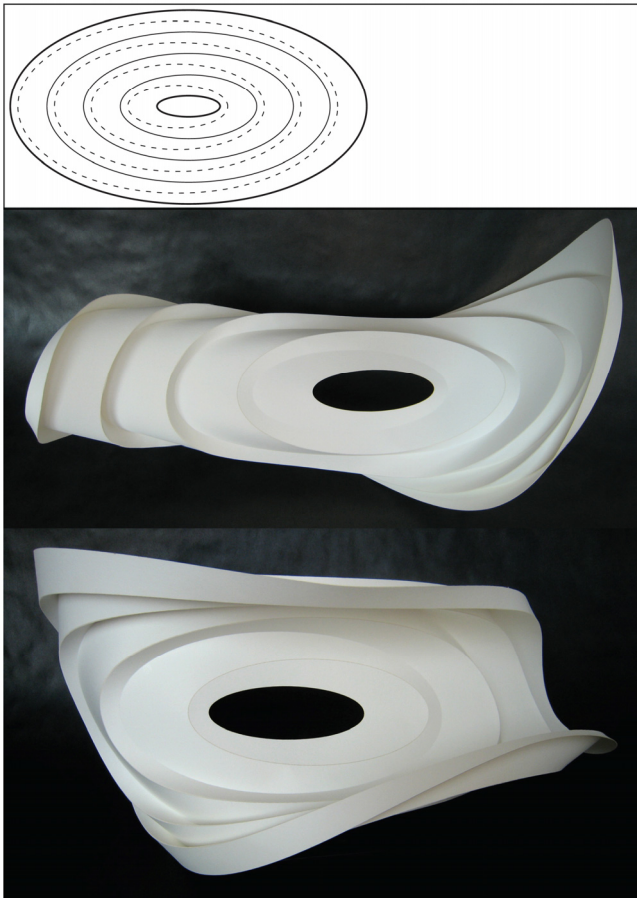


Figure 4c: ellipses; offset, center shifts left and right

A more negative example is the combination of three parabolas, shown in Figure 5, where it appears impossible to fold along all creases by a positive amount in the desired direction, resulting in a flat area. This outcome is not surprising, given the close proximity to a straight-crease design of concentric triangles, which behaves similarly. More interesting is that the closely related model shown in Figure 6, with two parabolas and a somewhat larger circular segment, folds nicely into a three-dimensional form with precisely the desired creases.

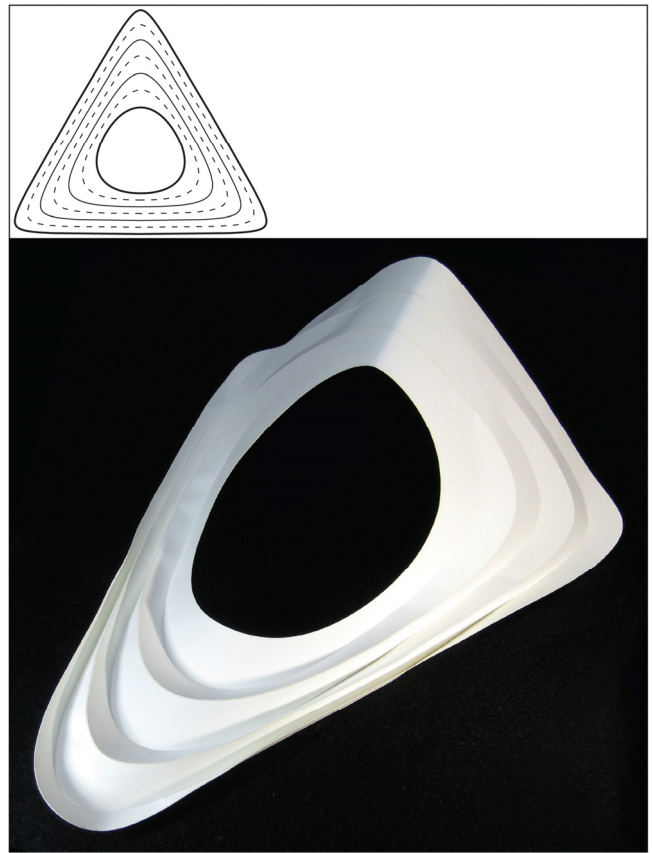


Figure 5: 3 parabolas; scaled, rotated

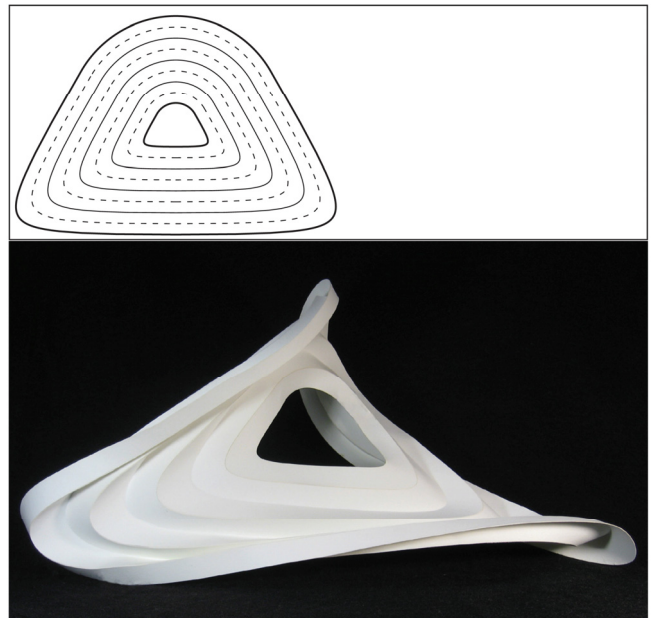


Figure 6: 2 parabolas & circle; scaled; midpt. in center

#### 4 Industry Context: Proof of Concept

The second part of this research is to investigate manufacturing techniques within an industry context, as related to the fabrication of architectural elements. We produced several prototypes for proof of concept and Figure 7 documents the successful ones. The goal is to create a direct connection from mathematical origami to fabrication technology relevant to architecture today.

The challenge regarding an architectural implementation is to find elastic materials that fold into these natural shapes, without showing additional creases, while being suitable for exterior applications. The proposed fabrication method is based on perforations, because a series of small holes can act as a guide for bending. S-shaped dashes for these perforations help metals bend easily [Ori]. Our successful experiments shown in Figure 7 were made of polycarbonate and steel cut with a water jet. This method also seems very promising for thicker sheets of aluminum.

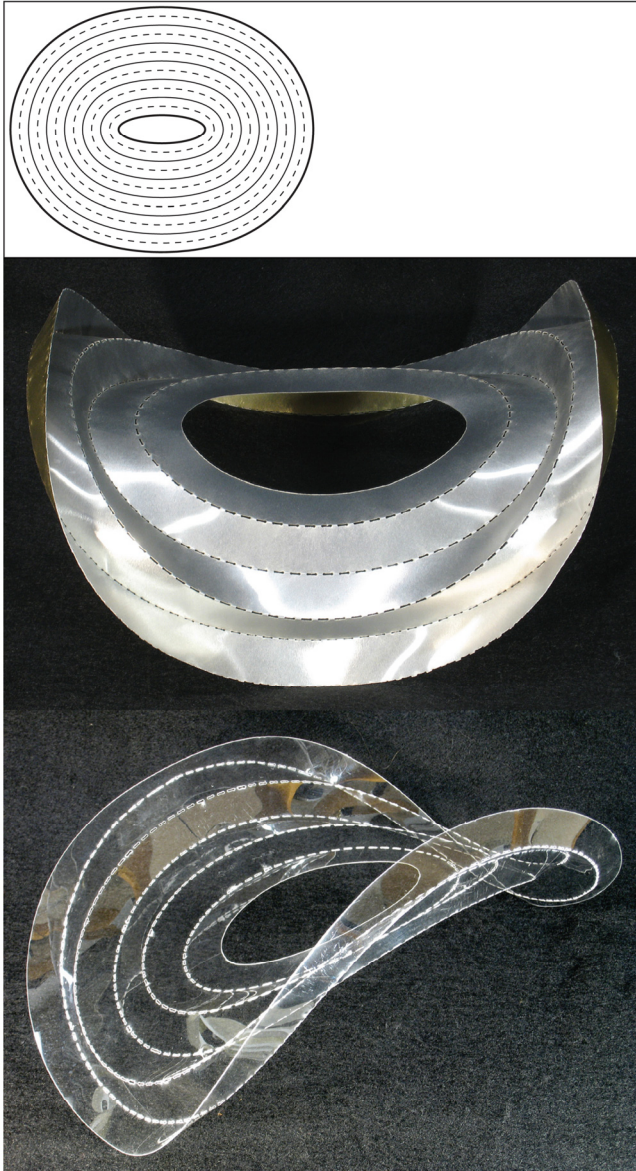


Figure 7: ellipses; offset w/ midpoint in center metal & polycarbonate

## Conclusion

This experimental research aims to elucidate the relationship between curved crease patterns and the natural three-dimensional forms that result. As little is known about this relationship, our trial-and-error approach may help indicate interesting behaviors that can be exploited in a more general algorithmic approach.

Creating three-dimensional shapes out of flat sheet goods has inherent architectural advantages and will contribute to the field

by providing form generation techniques for developable surfaces. We find this area ripe for further collaboration between mathematics, architecture, design, and fabrication.

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