Fun with Fonts: Algorithmic Typography

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Abstract. Over the past decade, we have designed five typefaces based on mathematical theorems and open problems, specifically computational geometry. These typefaces expose the general public in a unique way to intriguing results and hard problems in hinged dissections, geometric tours, origami design, physical simulation, and protein folding. In particular, most of these typefaces include puzzle fonts, where reading the intended message requires solving a series of puzzles which illustrate the challenge of the underlying algorithmic problem.

1 Introduction

Scientists use fonts every day to express their research through the written word. But what if the font itself communicated (the spirit of) the research? What if the way text is written, and not just the text itself, engages the reader in the science?

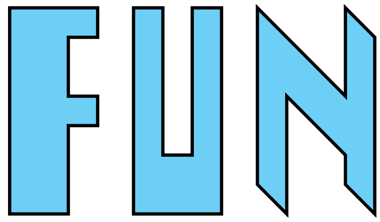
We have been designing a series of typefaces (font families) based on our computational geometry research. They are *mathematical typefaces* and *algorithmic typefaces* in the sense that they illustrate mathematical and algorithmic structures, theorems, and/or open problems. In all but one family, we include *puzzle typefaces* where reading the text itself requires engaging with those same mathematical structures. With a careful combination of puzzle and nonpuzzle variants, these typefaces enable the general public to explore the underlying mathematical structures and appreciate their inherent beauty, challenge, and fun.

This survey reviews the five typefaces we have designed so far, in chronological order. We describe each specific typeface design along with the underlying algorithmic field. Figure 1 shows the example of "FUN" written in all five typefaces. Anyone can experiment with writing text (and puzzles) in these typefaces using our free web applications.[1]

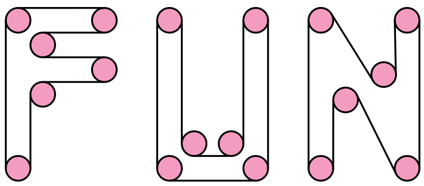
2 Hinged Dissections

A *hinged dissection* is a hinged chain of blocks that can fold into multiple shapes. Although hinged dissections date back over 100 years [Fre97], it was only very

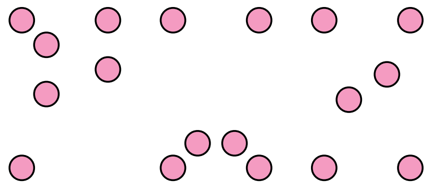
[1] http://erikdemaine.org/fonts/



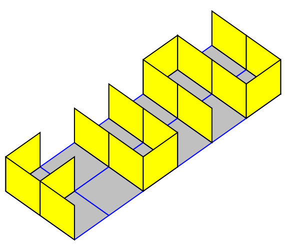
(a) Hinged-dissection typeface



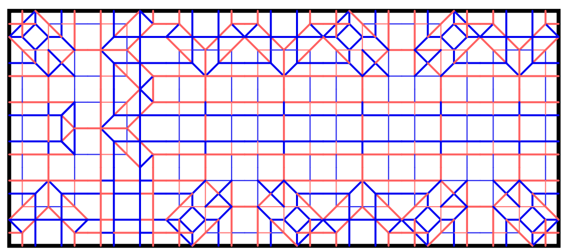
(b) Conveyer typeface, solved with belt



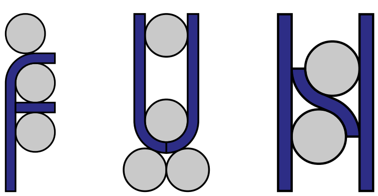
(c) Conveyer typeface, puzzle without belt



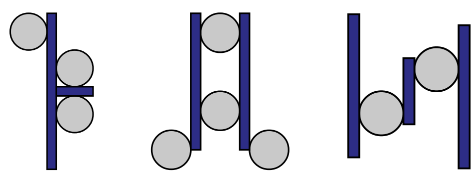
(d) Origami-maze typeface, 3D extrusion



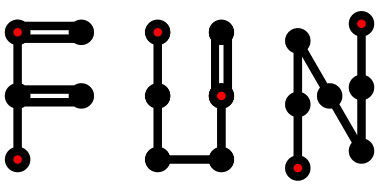
(e) Origami-maze typeface, puzzle crease pattern



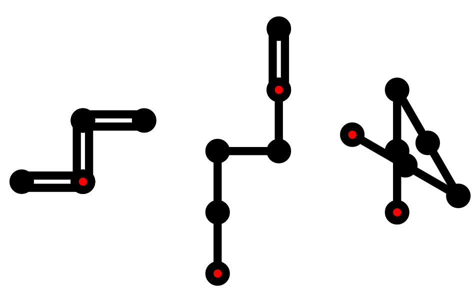
(f) Glass-squishing typeface, line art after squish



(g) Glass-squishing typeface, puzzle line art before squish



(h) Linkage typeface, correct font



(i) Linkage typeface, a puzzle font

Fig. 1: FUN written in all five of our mathematical typefaces.

Fig. 2: Hinged dissection typeface, from [DD03].

recently that we proved that hinged dissections exist, for any set of polygons of equal area [AAC+12]. That result was the culmination of many years of exploring the problem, starting with a theorem that any *polyform*—n identical shapes joined together at corresponding edges—can be folded from one universal chain of blocks (for each n) [DDEF99,DDE+05].

Our first mathematical/algorithmic typeface, designed in 2003 [DD03],[2] illustrates both this surprising way to hinge-dissect exponentially many polyform shapes, and the general challenge of the then-open hinged-dissection problem. As shown in Figure 2, we designed a series of glyphs for each letter and numeral as 32-abolos, that is, edge-to-edge gluings of 32 identical right isosceles triangles (half unit squares). In particular, every glyph has the same area. Applying our theorem about hinged dissections of polyforms [DDEF99,DDE+05] produces the 128-piece hinged dissection shown in Figure 3. This universal chain of blocks can fold into any letter in Figure 2, as well as a 4×4 square as shown in Figure 3.

An interesting open problem about this font is whether the chain of 128 blocks can be folded continuously without self-intersection into each of the glyphs. In

[2] http://erikdemaine.org/fonts/hinged/

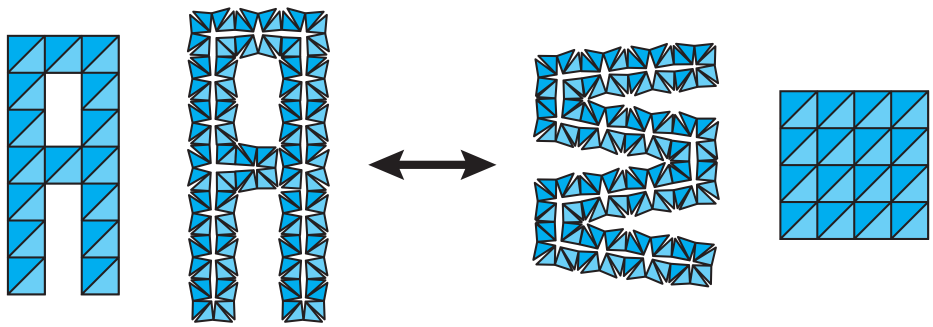


Fig. 3: Foldings of the 128-piece hinged dissection into the letter A and a square, from [DD03].

general, hinged chains of triangles can lock [CDD+10]. But if the simple structure of this hinged dissection enables continuous motions, we could make a nice animated font, where each letter folds back and forth between the informationless open chain (or square) and its folded state as the glyph. Given a physical instantiation of the chain (probably too large to be practical), each glyph is effectively a puzzle to see whether it can be folded continuously without self-intersection.

It would also be interesting to make a puzzle font within this typeface. Unfolded into a chain, each letter looks the same, as the hinged dissection is universal. We could, however, annotate the chain to indicate which parts touch which parts in the folded state, to uniquely identify each glyph (after some puzzling).

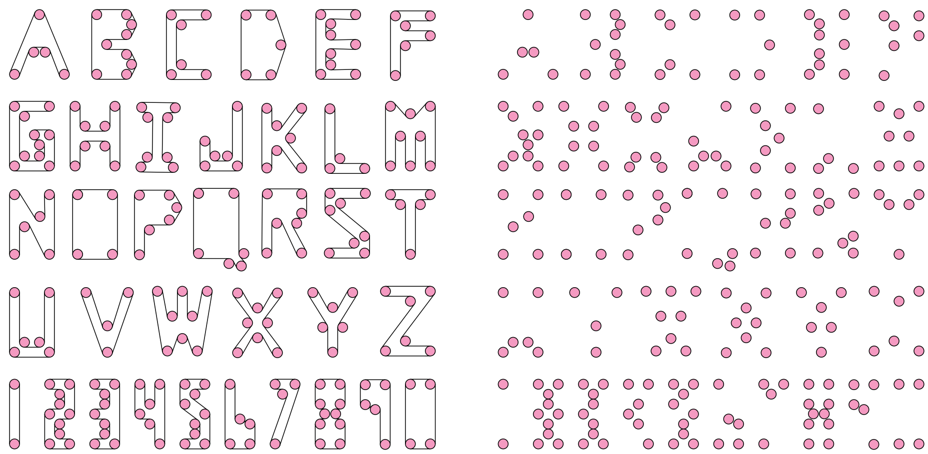
3 Conveyer Belts

A seemingly simple yet still open problem posed by Manual Abellanas in 2001 [Abe08] asks whether every disjoint set of unit disks (gears or wheels) in the plane can be visited by a single taut non-self-intersecting conveyer belt. Our research with Belén Palop first attempted to solve this problem, and then transformed into a new typeface design [DDP10a] and then puzzle design [DDP10b].

The conveyer-belt typeface, shown in Figure 4, consists of all letters and numerals in two main fonts.[3] With both disks and a valid conveyer belt (Figure 4(a)), the font is easily readable. But with just the disks (Figure 4(b)), we obtain a puzzle font where reading each glyph requires solving an instance of the open problem. (In fact, each distinct glyph needs to be solved only once, by recognizing repeated disk configurations.) Each disk configuration has been designed to have only one solution conveyer belt that looks like a letter or numeral, which implies a unique decoding.

The puzzle font makes it easy to generate many puzzles with embedded secret messages [DDP10b]. By combining glyphs from both the puzzle and solved (belt)

[3] http://erikdemaine.org/fonts/conveyer/



(a) With belts. (b) Without belts.

Fig. 4: Conveyer belt alphabet, from [DDP10a].

font, we have also designed a series of puzzle/art prints. Figure 5 shows a self-referential puzzle/art print which describes the very open problem on which it is based.

4 Origami Mazes

In computational origami design, the typical goal is to develop algorithms that fold a desired 3D shape from the smallest possible rectangle of paper of a desired aspect ratio (typically a square). One result which achieves a particularly efficient use of paper is *maze folding* [DDK10a]: any 2D grid graph of horizontal and vertical integer-length segments, extruded perpendicularly from a rectangle of paper, can be folded from a rectangle of paper that is a constant factor larger than the target shape. A striking feature is that the scale factor between the unfolded piece of paper and the folded shape is independent of the complexity of the maze, depending only on the ratio of the extrusion height to the maze tunnel width. (For example, a extrusion/tunnel ratio of 1 : 1 induces a scale factor of 3 : 1 for each side of the rectangle.)

The origami-maze typeface, shown in Figure 6, consists of all letters in three main fonts [DDK10b].[4] In the 2D font (Figure 6(a)), each glyph is written as a 2D grid graph before extrusion. In the 3D font (Figure 6(b)), each glyph is drawn as a 3D extrusion out of a rectangular piece of paper. In the crease-pattern font (Figure 6(c)), each glyph is represented by a crease pattern produced by the maze-folding algorithm, which folds into the 3D font. By properties of the algorithm, the crease-pattern font has the feature that glyphs can be attached

[4] http://erikdemaine.org/fonts/maze/

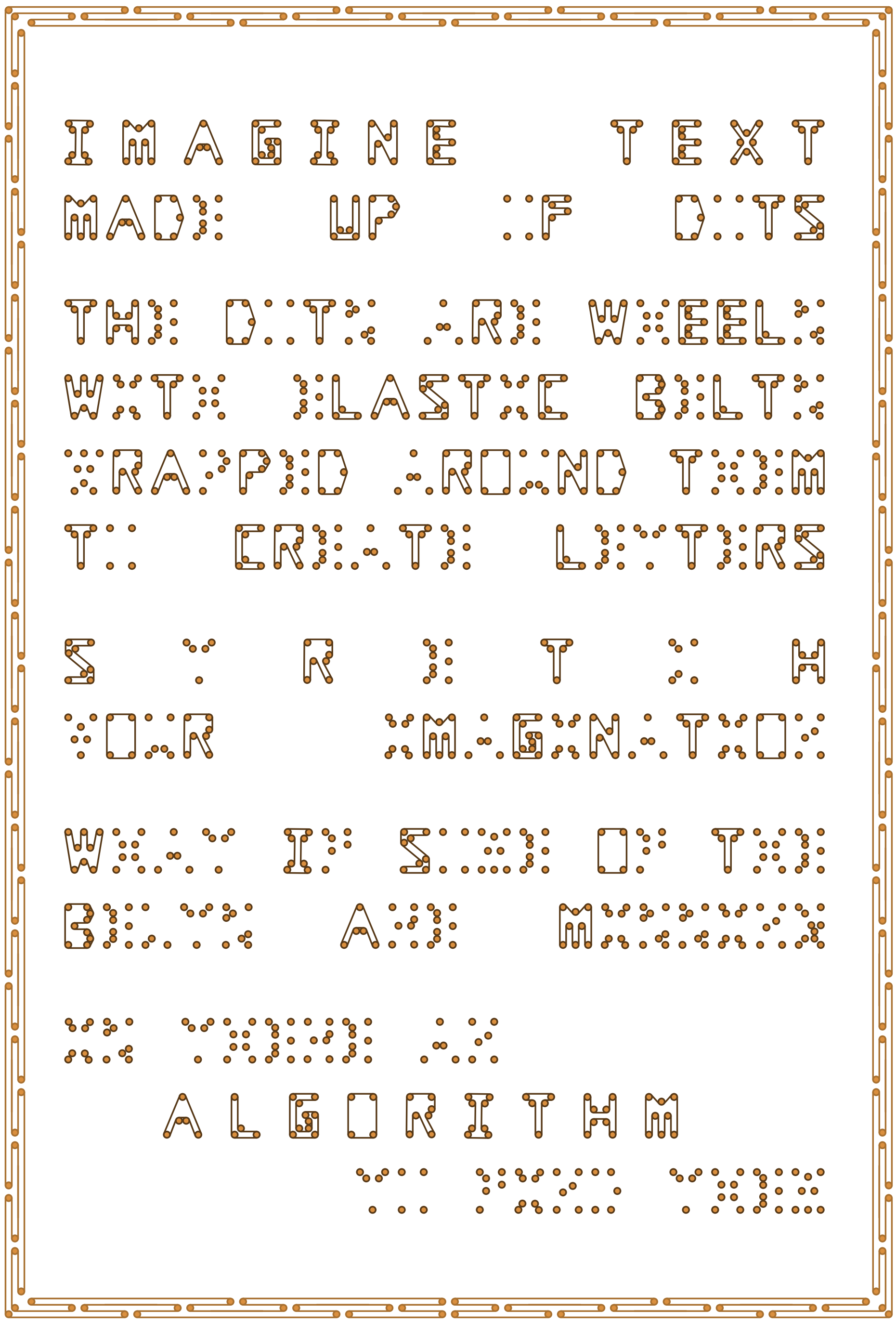
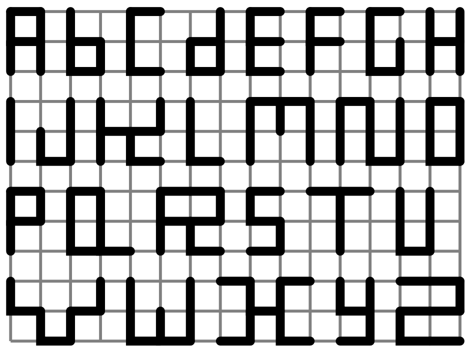
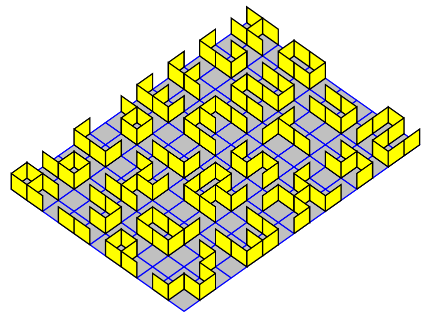


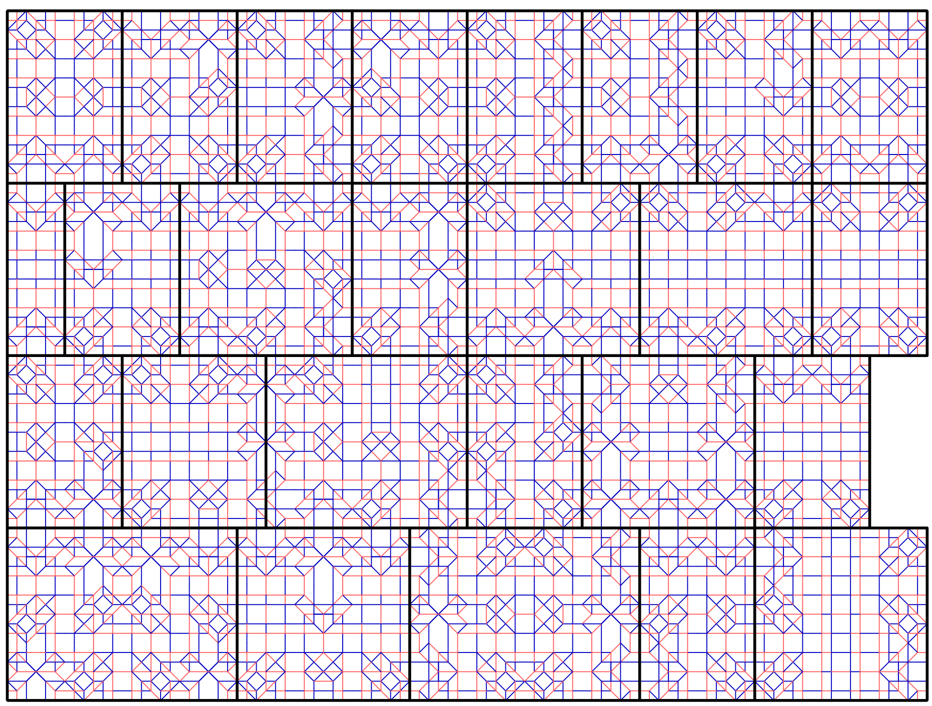
Fig. 5: "Imagine Text" (2013), limited-edition print, Erik D. Demaine and Martin L. Demaine, which premiered at the Exhibition of Mathematical Art, Joint Mathematics Meetings, San Diego, January 2013.



(a) 2D grid maze



(b) 3D extrusion



(c) Crease pattern

Fig. 6: Origami-maze typeface, from [DDK10b]: (c) folds into (b), which is an extrusion of (a). Dark lines are mountain folds; light lines are valley folds; bold lines delineate letter boundaries and are not folds.

together on their boundary to form a larger crease pattern that folds into all of the letters as once. For example, the entire crease pattern of Figure 6(c) folds into the 3D shape given by Figure 6(b).

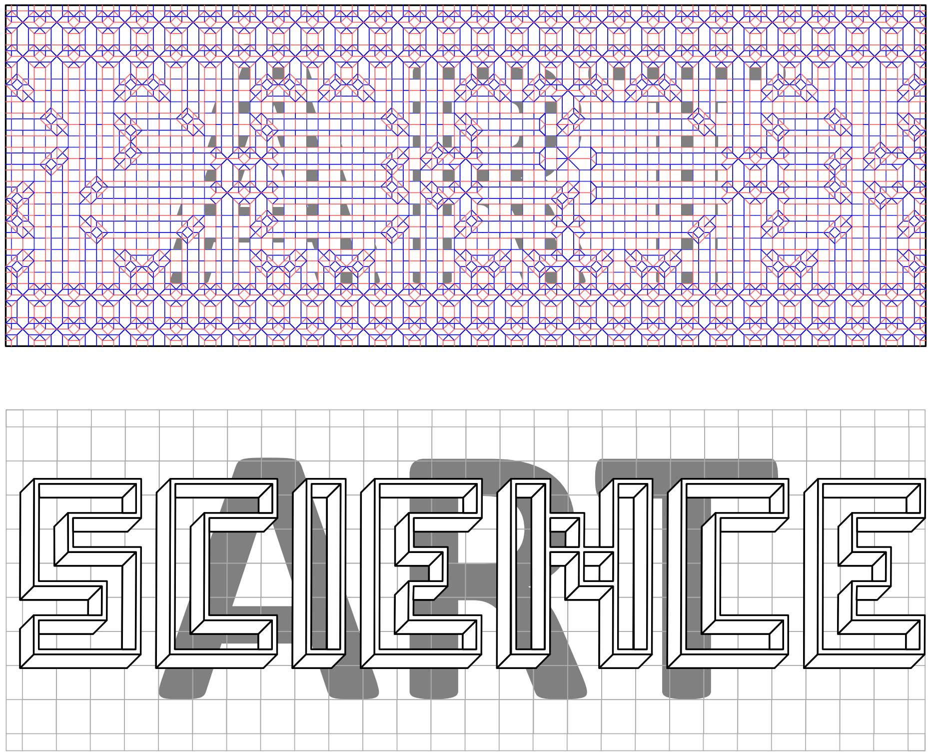


Fig. 7: “Science/Art” (2011), limited-edition print, Erik D. Demaine and Martin L. Demaine, which premiered at the Exhibition of Mathematical Art, Joint Mathematics Meetings, Boston, January 2012.

The crease-pattern font is another puzzle font: each glyph can be read by folding, either physically or in your head. With practice, it is possible to recognize the extruded ridges from the crease pattern alone, and devise the letters in the hidden message. We have designed several puzzles along these lines [DDK10b].

It is also possible to overlay a second puzzle within the crease-pattern font, by placing a message or image in the ground plane of the 3D folded shape, dividing up by the grid lines, and unfolding those grid cells to where they belong in the crease pattern. Figure 7 shows one print design along these lines, with the crease pattern defining the 3D extrusion of “SCIENCE” while the gray pattern comes together to spell “ART”. In this way, we use our typeface design to inspire new print designs.

5 Glass Squishing

Glass blowing is an ancient art form, and today it uses most of the same physical tools as centuries ago. In computer-aided glass blowing, our goal is to harness geometric and computational modeling to enable design of glass sculpture and prediction of how it will look ahead of time on a computer. This approach enables extensive experimentation with many variations of a design before committing the time, effort, and expense required to physically blow the piece. Our free software Virtual Glass [WBM+12] currently focuses on computer-aided design of the highly geometric aspects of glass blowing, particularly glass cane.

One aspect of glass blowing not currently captured by our software is the ability to "squish" components of glass together. This action is a common technique for combining multiple glass structures, in particular when designing elaborate glass cane. To model this phenomenon, we need a physics engine to simulate the idealized behavior of glass under "squishing".

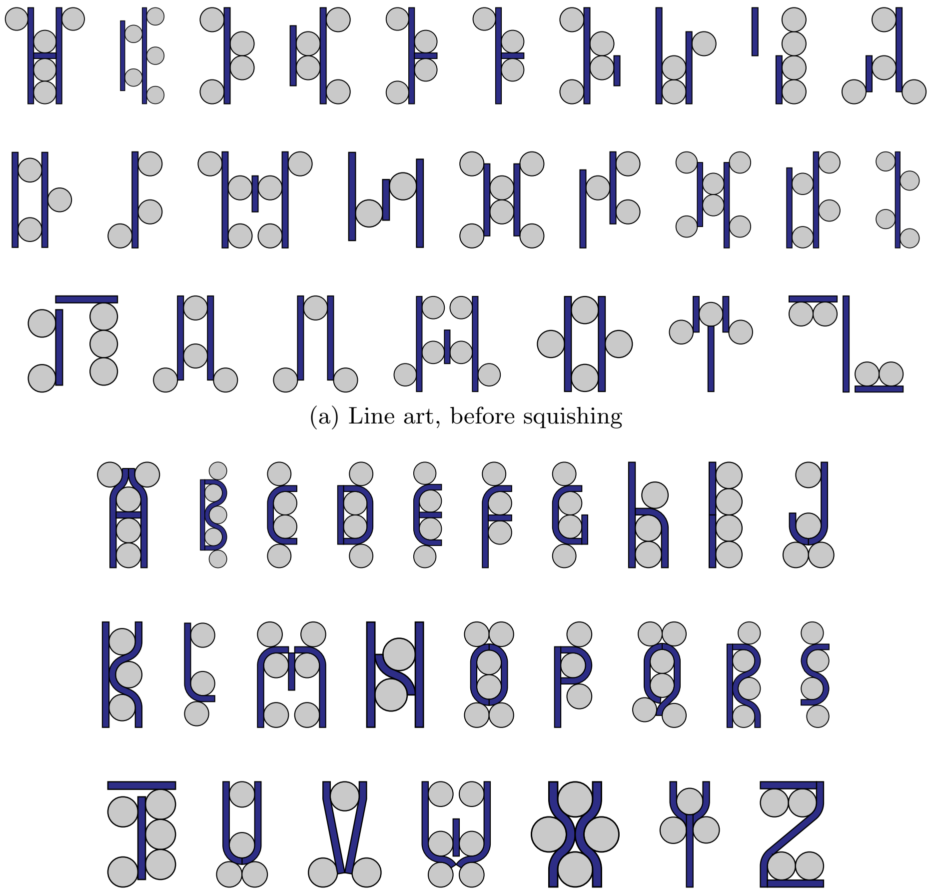
To better understand this physical behavior, we designed a glass-squishing typeface during a 2014 residency at Penland School of Crafts. As shown in Figure 8, we designed arrangements of simple glass components—clear disks and opaque thin lines/cylinders—that, when heated to around 1400°F and squished between two vertical steel bars, produce any desired letter. The typeface consists of five main fonts: photographs of the arrangements before and after squishing, line drawings of these arrangements before and after squishing, and video of the squishing process. The "before" fonts are puzzle fonts, while the "after" fonts are clearly visible. The squishing-process font is a rare example of a video font, where each glyph is a looping video. Figure 9 shows stills from the video for the letters F-U-N. See the web app for the full experience.[5]

Designing the before-squishing glass arrangements required extensive trial and error before the squished result looked like the intended glyph. This experimentation has helped us define a physical model for the primary forces and constraints for glass squishing in 2D, which can model the cross-section of 3D hot glass. We plan to implement this physical model to both create another video font of line art simulating the squishing process, and to enable a new category of computer-aided design of blown glass in our Virtual Glass software. In this way, we use typeface design to experiment with and inform our computer science research.

6 Fixed-Angle Linkages

Molecules are made up of atoms connected together by bonds, with bonds held at relatively fixed lengths, and incident bonds held at relatively fixed angles. In mathematics, we can model these structures as *fixed-angle linkages*, consisting of rigid bars (segments) connected at their endpoints, with specified fixed lengths for the bars and specified fixed angles between incident bars. A special case of particular interest is a *fixed-angle chain* where the bars are connected together in a path, which models the backbone of a protein. There is extensive algorithmic research on fixed-angle chains and linkages, motivated by mathematical models of protein folding; see, e.g., [DO07, chapters 8–9]. In particular, the literature has studied *flat states* of fixed-angle chains, where all bars lie in a 2D plane.

[5] http://erikdemaine.org/fonts/squish/



(a) Line art, before squishing

(b) Line art, after squishing

Fig. 8: Glass-squishing typeface.

Fig. 9: Frames from the video font rendering of F-U-N.

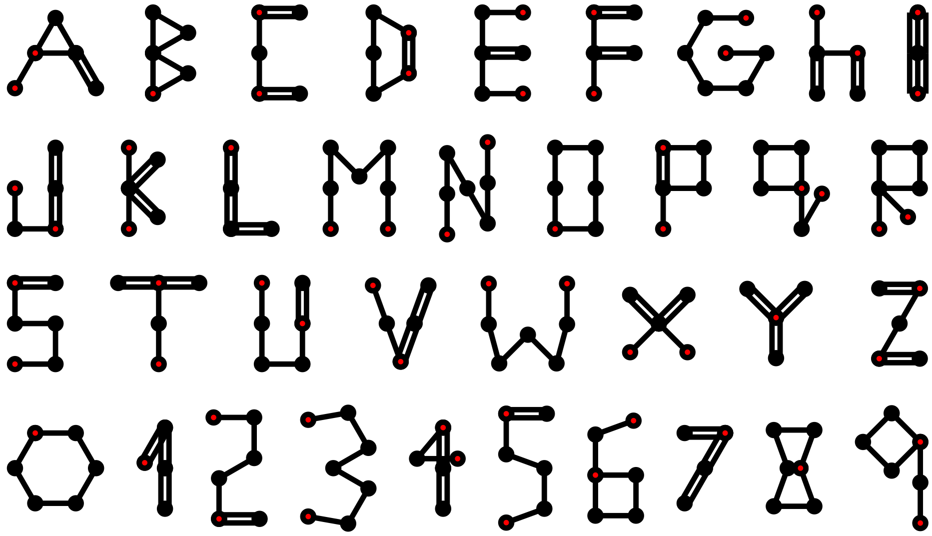


Fig. 10: Linkage typeface, from [DD14]. Each letter has several glyphs; shown here is the "correct" glyph. Doubled and tripled edges are spread apart for easier visibility.

Our linkage typeface, shown in Figure 10, consists of a fixed-angle chain for each letter and numeral. Every fixed-angle chain consists of exactly six bars, each of unit length. Hence, each chain is defined just by a sequence of five measured (convex) angles. Each chain, however, has many flat states, depending on whether the convex side of each angle is on the left or the right side of the chain. Thus, each chain has $2^5 = 32$ glyphs depending on the choice for each of the five angles. (In the special cases of zero and 360° angles, the choice has no effect so the number of distinct glyphs is smaller.)

Thus each letter and numeral has several possible glyphs, only a few of which are easily recognizable; the rest are puzzle glyphs. Figure 11 shows some example glyphs for F-U-N. We have designed the fixed-angle chains to be uniquely decodable into a letter or numeral; the incorrect foldings do not look like another letter or numeral. The result is a random puzzle font.[6] Again we have used this font to design several puzzles [DD14].

In addition, there is a rather cryptic puzzle font given just by the sequence of angles for each letter. For example, F-U-N can be written as 90-0-90-90-0 0-180-90-90-180 180-30-180-30-180.

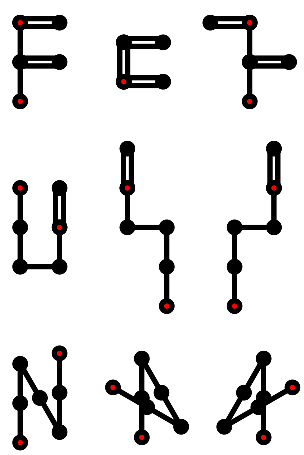


Fig. 11: A few random linkage glyphs for F-U-N.

[6] http://erikdemaine.org/fonts/linkage/

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