HINGED DISSECTION OF THE ALPHABET

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Abstract

We present one hinged dissection of a square that can be folded into every letter of a polyabolo alphabet.

Hinged dissections. Hinged dissection is a particular form of dissection in which one shape is sliced into pieces and hinged at their vertices so that the mechanism can be folded into another shape or shapes. The classic example is the hinged dissection of an equilateral triangle to a square described by Dudeney and shown in Figure 1; see [1] for the history of this dissection, as well as some other hinged dissections. Recently, Frederickson [2] has written a book devoted entirely to hinged dissections and techniques for designing them.

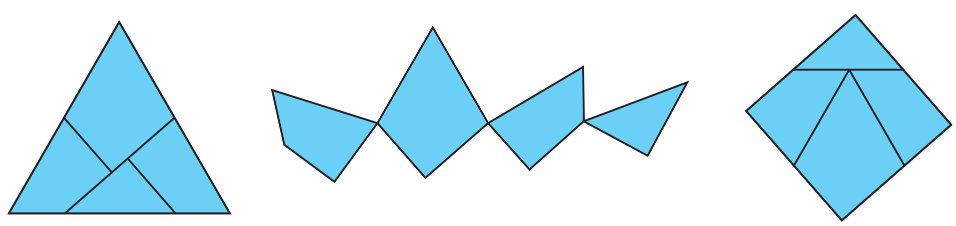
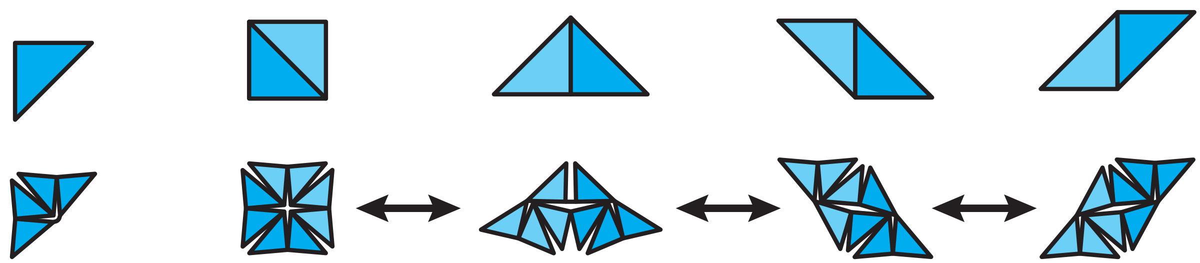


Figure 1: Hinged dissection of an equilateral triangle into a square [1].

Theory. An intriguing open problem is whether every pair of polygons has a hinged dissection. Recently, there have been two interesting results addressing this problem. Eppstein [3] showed how to hinge-dissect any polygon into its mirror image. Eppstein, Frederickson, Friedman, and the present authors [4] demonstrated a wide family of hinged dissections for *polyforms*, that is, shapes made up of repeated copies of a common polygon glued together edge-to-edge. Examples of polyforms include polyominoes, polyiamonds, polyhexes, and polyaboloes. The hinged dissections of [4] are *cycles* (closed chains) of pieces, which is even more restrictive than the standard open chains of pieces as in Figure 1.

Polyaboloes. Here we focus on polyforms called *polyaboloes*, which are made up of congruent right isosceles triangles (half-squares). In this context, the basic idea of the hinged dissections in [4] is to split each right isosceles triangle into four subtriangles and hinge them together as shown in Figure 2(a). Then these four-piece hinged dissections of right isosceles triangles can be combined together edge-to-edge by breaking them at the midpoint of the common edge; see Figure 2(b) for the case of two triangles. In general, for an n-abolo, the hinged dissection has $4n$ pieces.



(a) Monabolo. (b) Diabolos.

Figure 2: Hinged dissections from [4] for small polyaboloes.

Alphabet. We have designed an alphabet in which every letter and digit is a 32-abolo, as shown in Figure 3. A 4×4 square is also a 32-abolo. Thus, from the results of [4], we obtain a single hinged dissection with 128 pieces that folds into all letters and digits and the 4×4 square. Two example foldings of this hinged dissection, one for the letter A and the other for the square, are shown in Figure 4.

Related work. Harry Lindgren [5] designed dissections of individual rectilinear letters to a square, specifically E, F, H, I, L, M, N, T, V, W, X, Y, and Z. To make the dissections similar, he decided that each letter would have a height five times the thickness of the strokes. Our work in some sense strengthens these results, both to hinged dissections and to all letters and digits.

Alphabet design. The letters and digits all have height 7 and area 16. The main motivation for these parameters is to make the area a square number, so that the hinged dissection could fold into an integral square. An area of 9 is too small for a good-looking alphabet (consider e.g. the digit 8 or letter B).

Figure 3: The 32-abolo alphabet.

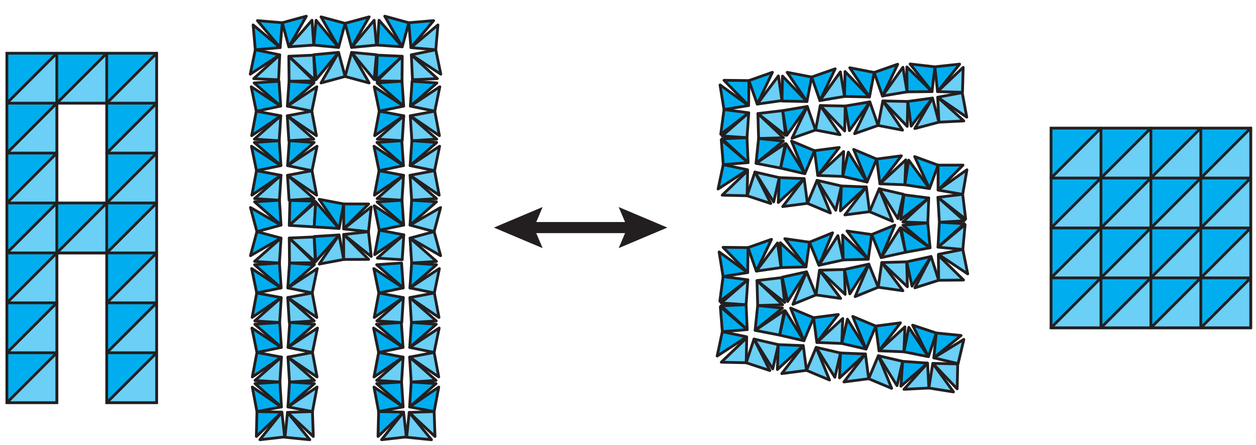


Figure 4: Foldings of the 128-piece hinged dissection into the letter A and a square.

References

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