LINKAGE PUZZLE FONT

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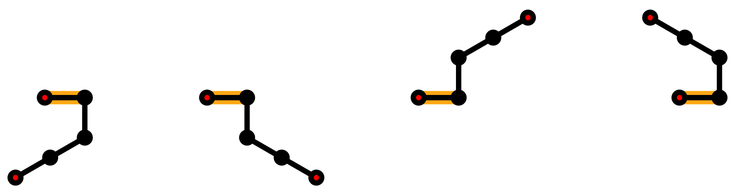
What do the following polygonal chains have in common?



In all four drawings, the sequence of vertex angles starting from the highlighted edge is 90°, 120°, 180°. The corresponding edge lengths match as well — they are all in fact 1. Up to rotation, these are all four drawings of this sequence of edge lengths and angles. Each drawing can be obtained from the previous drawing by *spinning* one edge: holding one half of the chain fixed, and letting the other half rotate by 180° around the edge.

In general, a *fixed-angle chain* is defined by a sequence of edge lengths and measured angles between consecutive edges. Here *measured angle* refers to the minimum of the two angles on either side of the vertex, which is always between 0 and 180° (as opposed to, say, the clockwise angle, which can be bigger). Equivalently, the measured angle can be viewed as the angle between the two edges viewed as line segments in 3D. A *configuration* of a fixed-angle chain is a polygonal chain that has the correct sequence of edge lengths and measured angles. (Here we consider configurations only in 2D, but configurations in 3D also make sense.) Edge spins, as described above, are the basic moves that a fixed-angle chain can make: any configuration can be transformed into any other by a sequence of edge spins.

Fixed-angle chains, and more generally fixed-angle linkages, have been studied extensively in the field of geometric folding algorithms; see, e.g., Chapters 8–9 of [DO07]. They arise naturally both to represent joint constraints in robotics and as geometric models of molecular chemistry and biology. In particular, a reasonable mechanical model of atoms in a molecule is as a fixed-angle linkage, and the backbone of a protein can be modeled as a fixed-angle chain. Thus, a protein folds approximately how a fixed-angle chain folds (in 3D).

We like to transform interesting mathematical constructions, theorems, and open problems into *mathematical fonts*.[1] Previous such fonts are based on a theorem about hinged dissections [DD03], an open problem about conveyer belts [DDP10a, DDP10b], and a theorem for designing origami mazes [DDK10]. The latter two fonts are also *puzzle fonts*: in one form, reading the written message is actually a puzzle. These infinite families of puzzles provide an accessible medium for understanding the mathematical principles that form the basis of the font, so that everyone can appreciate the beautiful challenge of the underlying construction, theorem, or open problem.

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[1]See http://erikdemaine.org/fonts/.

Here we describe a mathematical puzzle font based on fixed-angle linkages. For each letter and number, we designed a fixed-angle unit 6-chain, consisting of exactly six unit-length segments; see Figure 1. Thus each chain is determined entirely by a sequence of five measured angles. Using a computer program, we computed all $2^5 = 32$ possible 2D configurations of these fixed-angle chains, corresponding to all possible subsets of edge spins.

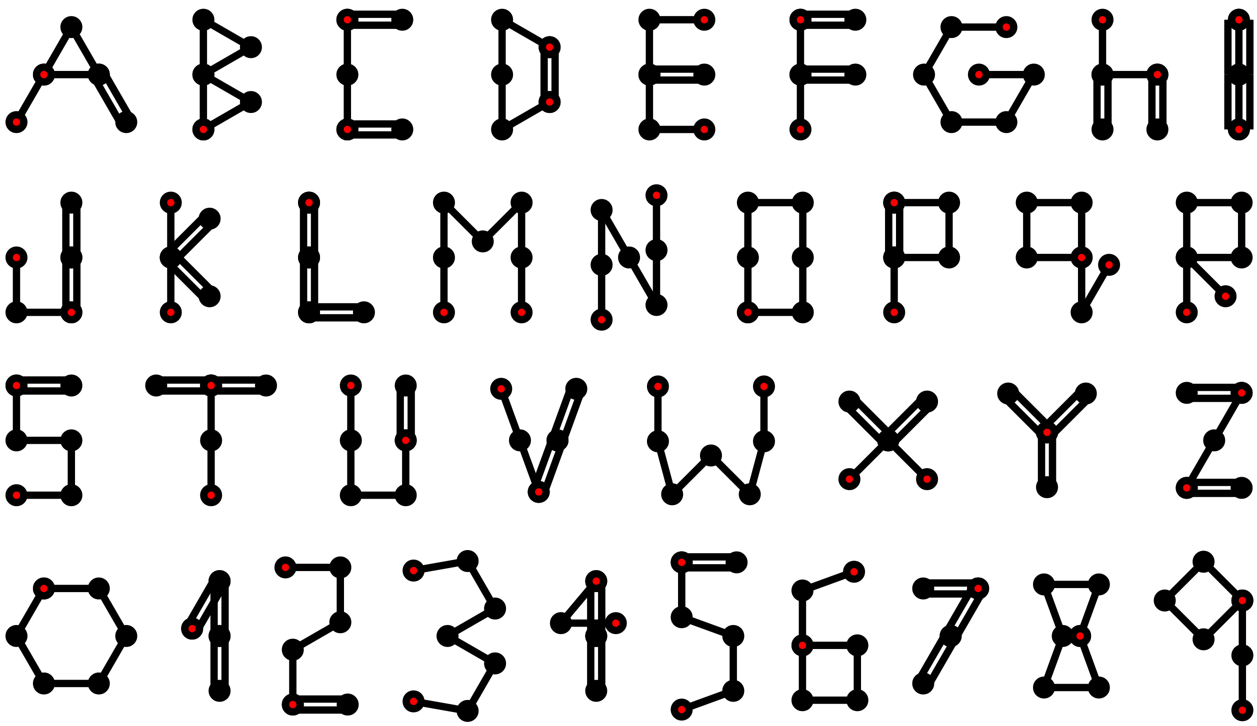


Figure 1: The complete linkage alphabet, in the intended configurations.

Each fixed-angle chain is designed to have a unique configuration that looks like a letter or number, and thus can be uniquely "read". This is the puzzle: we choose a random configuration of each letter, and the reader must flip the edges until finding a unique letter or number. Sometimes the chain traverses a segment twice, in which case we draw it as a doubled edge (or in the case of I, a tripled edge). The two ends of each chain (occasionally at the same place) are highlighted; this makes it possible to uniquely construct the sequence of angles underlying the fixed-angle chain given just one of its random configurations. (Knowing the end vertices is not enough to reconstruct the chain in general, but in our font except for I, the only ambiguity is when the chain forms a doubled cycle, in which case knowing the starting/ending vertex disambiguates.)

There are at least three different fonts based on these linkages. In the *solved font* (as in the title of this paper), each linkage is in the intended configuration, so the text can be read directly. In the *randomized font*, each linkage is in a random configuration (occasionally the intended one), and it is a puzzle to reconstruct the text. In the *angle code font*, we just give the sequence of measured angles for each chain, making for an even more challenging puzzle to reconstruct the text.

In the following pages, we provide a series of randomized font puzzles with hidden messages. These puzzles were created using a freely available web application,[2] which you can use to generate your own puzzles.

[2] http://erikdemaine.org/fonts/linkage/

Puzzle Solutions:

1. Martin Gardner • 2. G4G11 • 3. read me
4. protein folding • 5. hello world • 6. have fun

References

[DD03] Erik D. Demaine and Martin L. Demaine. Hinged dissection of the alphabet. *Journal of Recreational Mathematics*, 31(3):204–207, 2003.

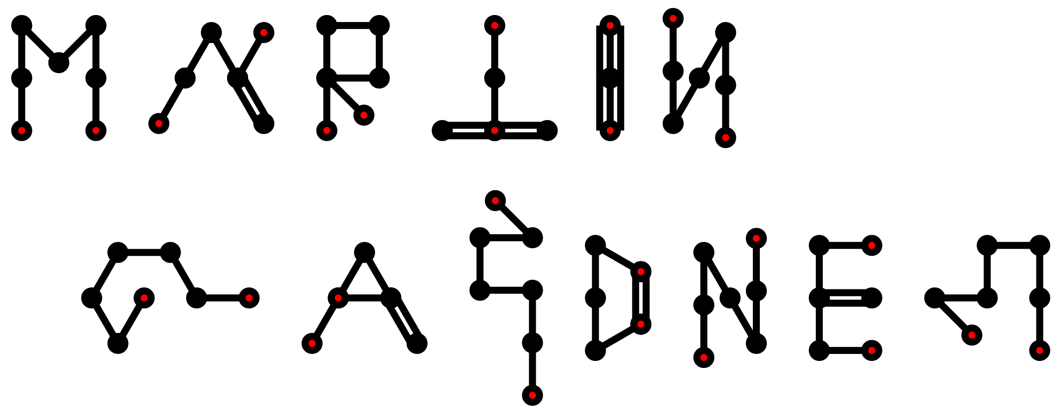
[DDK10] Erik D. Demaine, Martin L. Demaine, and Jason Ku. Origami maze puzzle font. In *Proceedings of the 9th Gathering for Gardner*, 2010.

[DDP10a] Erik D. Demaine, Martin L. Demaine, and Belén Palop. Conveyer-belt alphabet. In Pars Foundation, editor, *Findings on Elasticity*. Lars Müller Publishers, 2010.

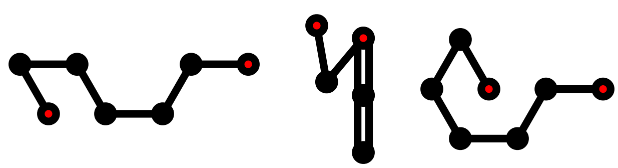
[DDP10b] Erik D. Demaine, Martin L. Demaine, and Belén Palop. Conveyer belt puzzle font. In *Proceedings of the 9th Gathering for Gardner*, 2010.

[DO07] Erik D. Demaine and Joseph O'Rourke. *Geometric Folding Algorithms: Linkages, Origami, Polyhedra*. Cambridge University Press, July 2007.

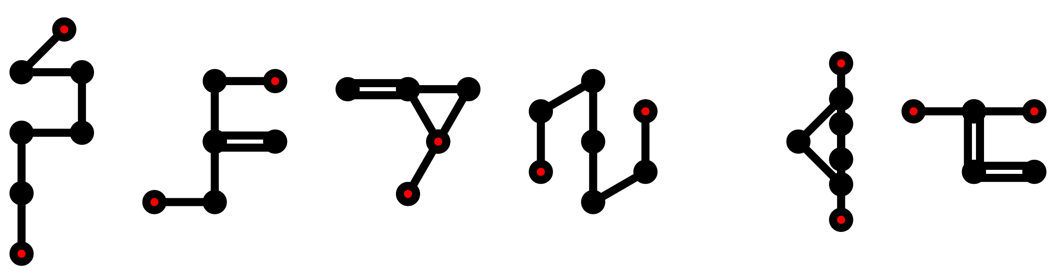
Puzzle 1.



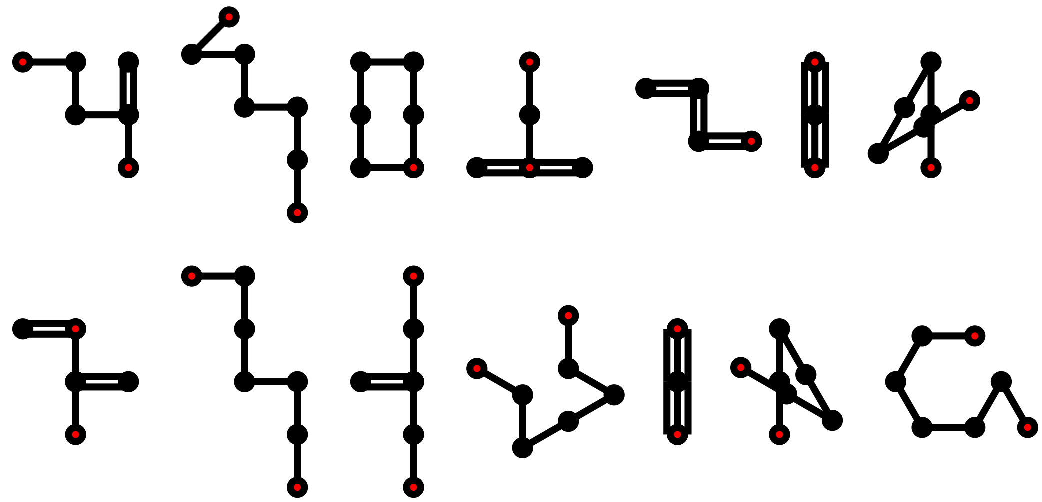
Puzzle 2.



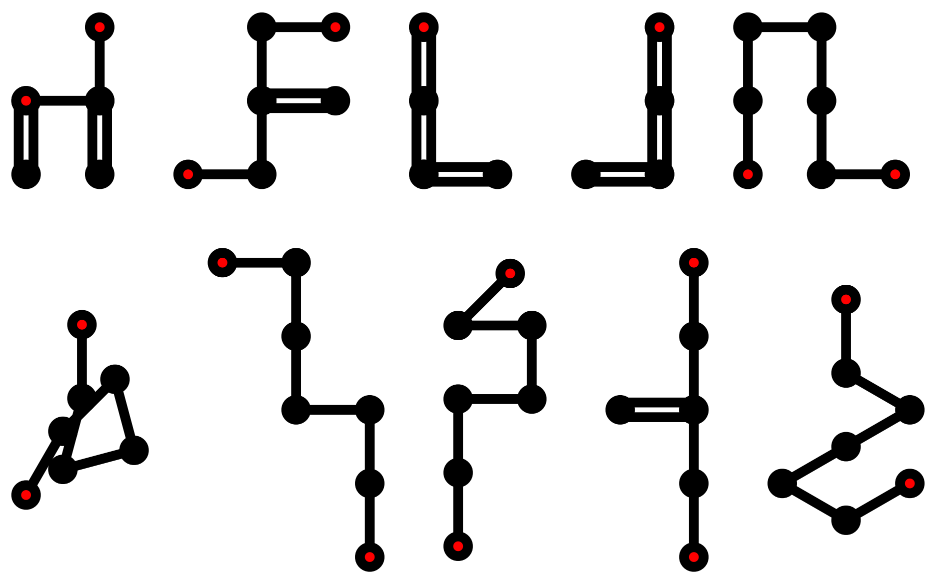
Puzzle 3.



Puzzle 4.



Puzzle 5.



Puzzle 6.