

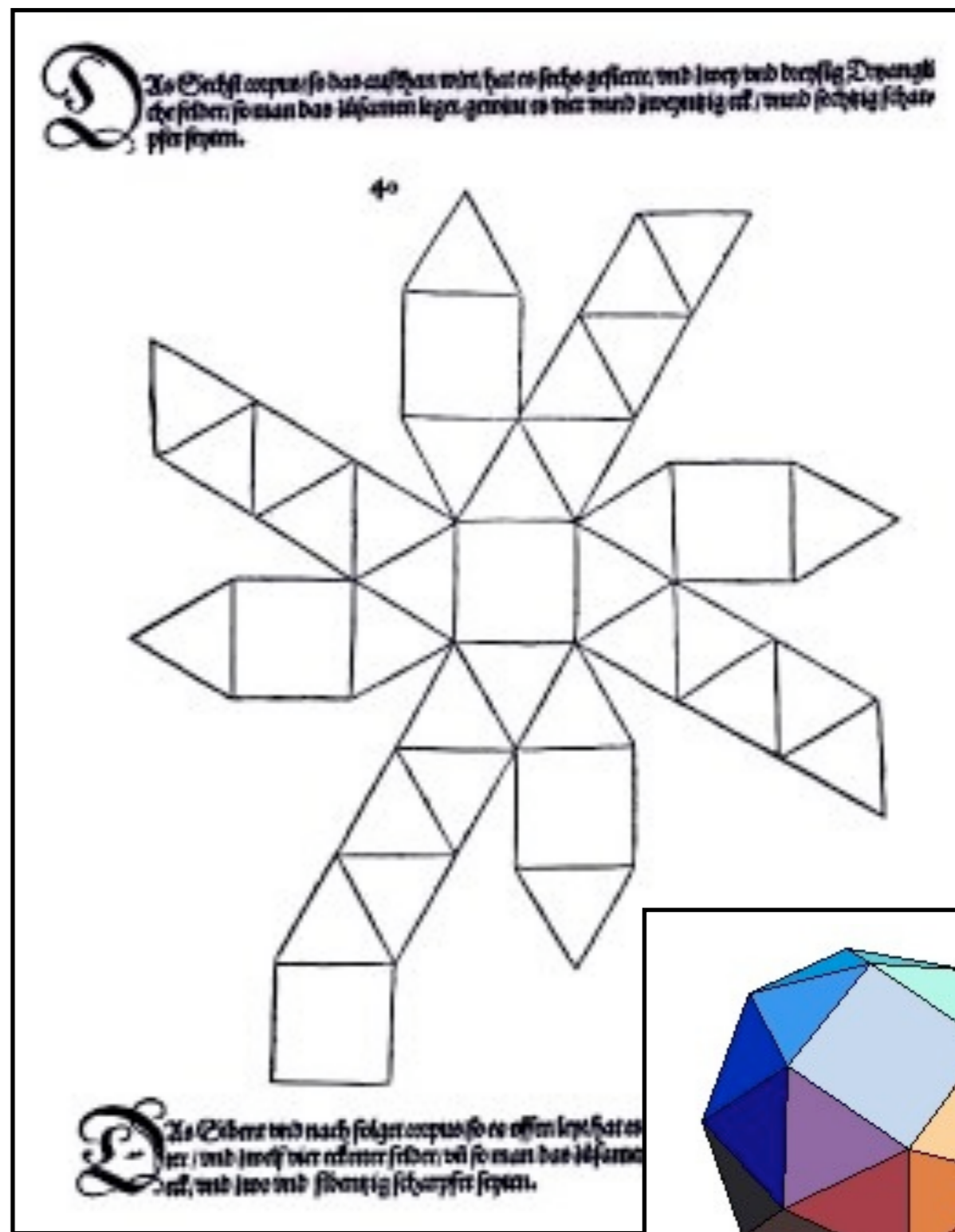
Zipper Unfoldings of Polyhedral Complexes

Erik Demaine Martin Demaine
Anna Lubiw Arlo Shallit Jonah Shallit

Unfolding Polyhedra—Durer 1400's



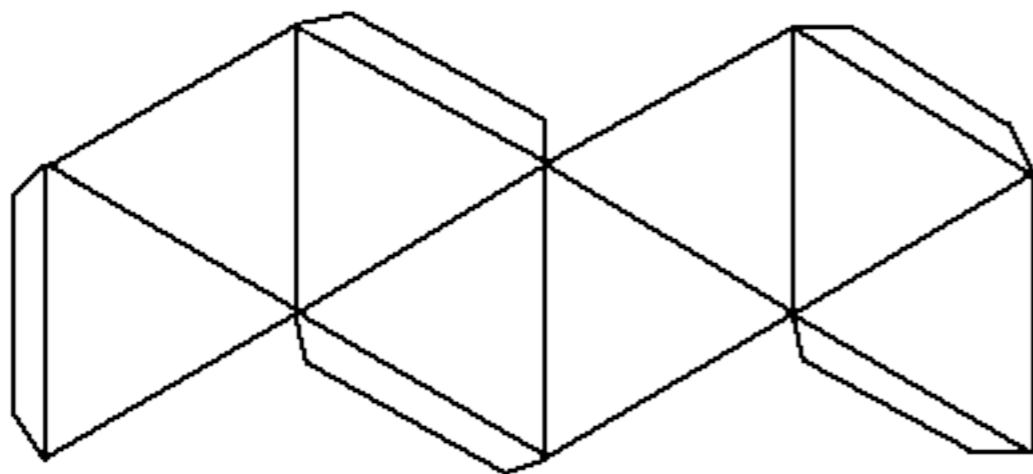
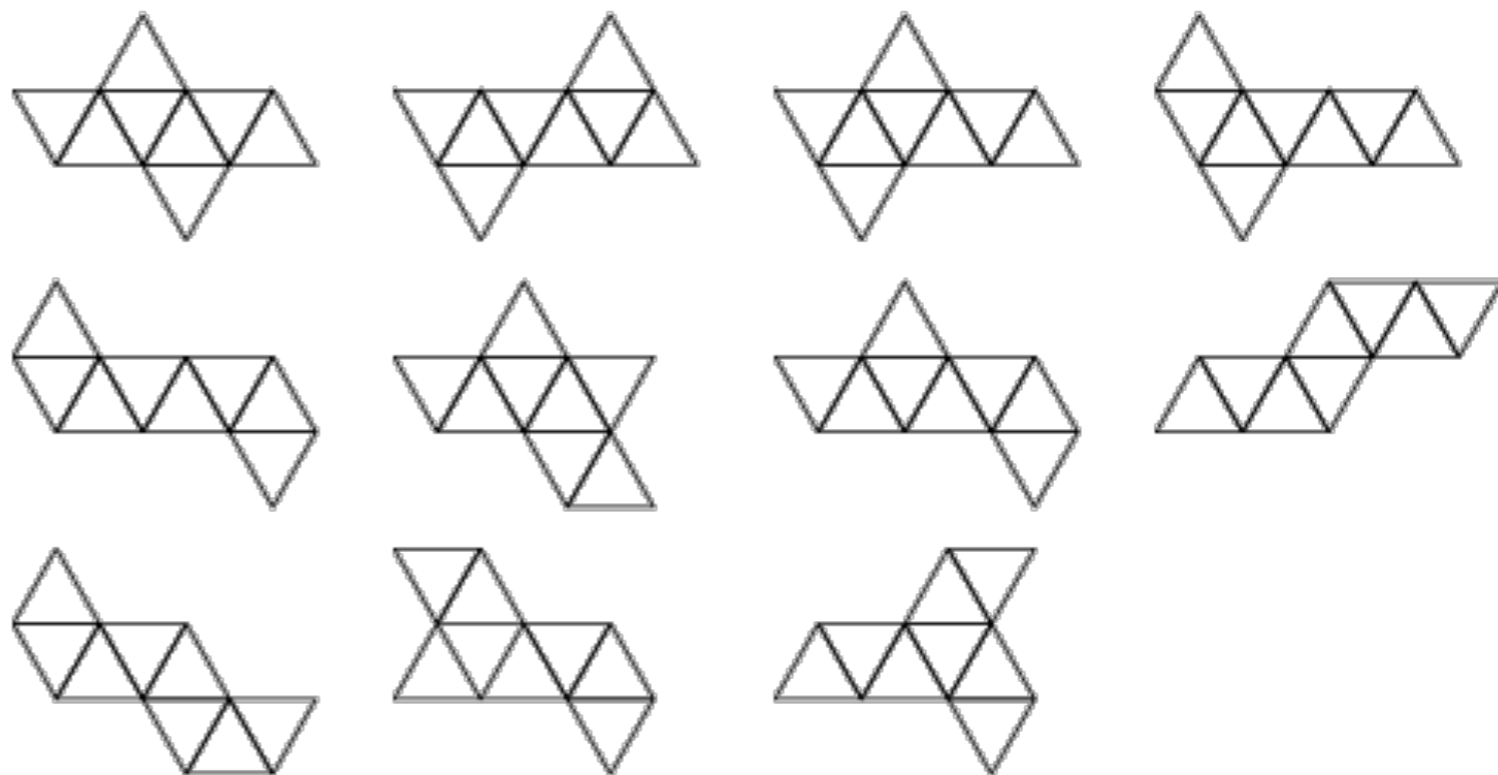
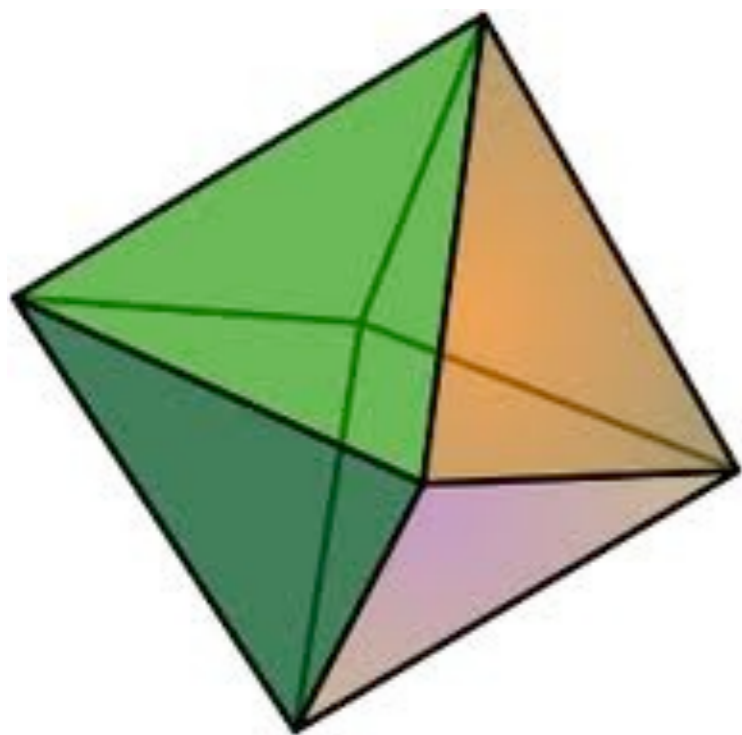
Durer, 1498



snub cube

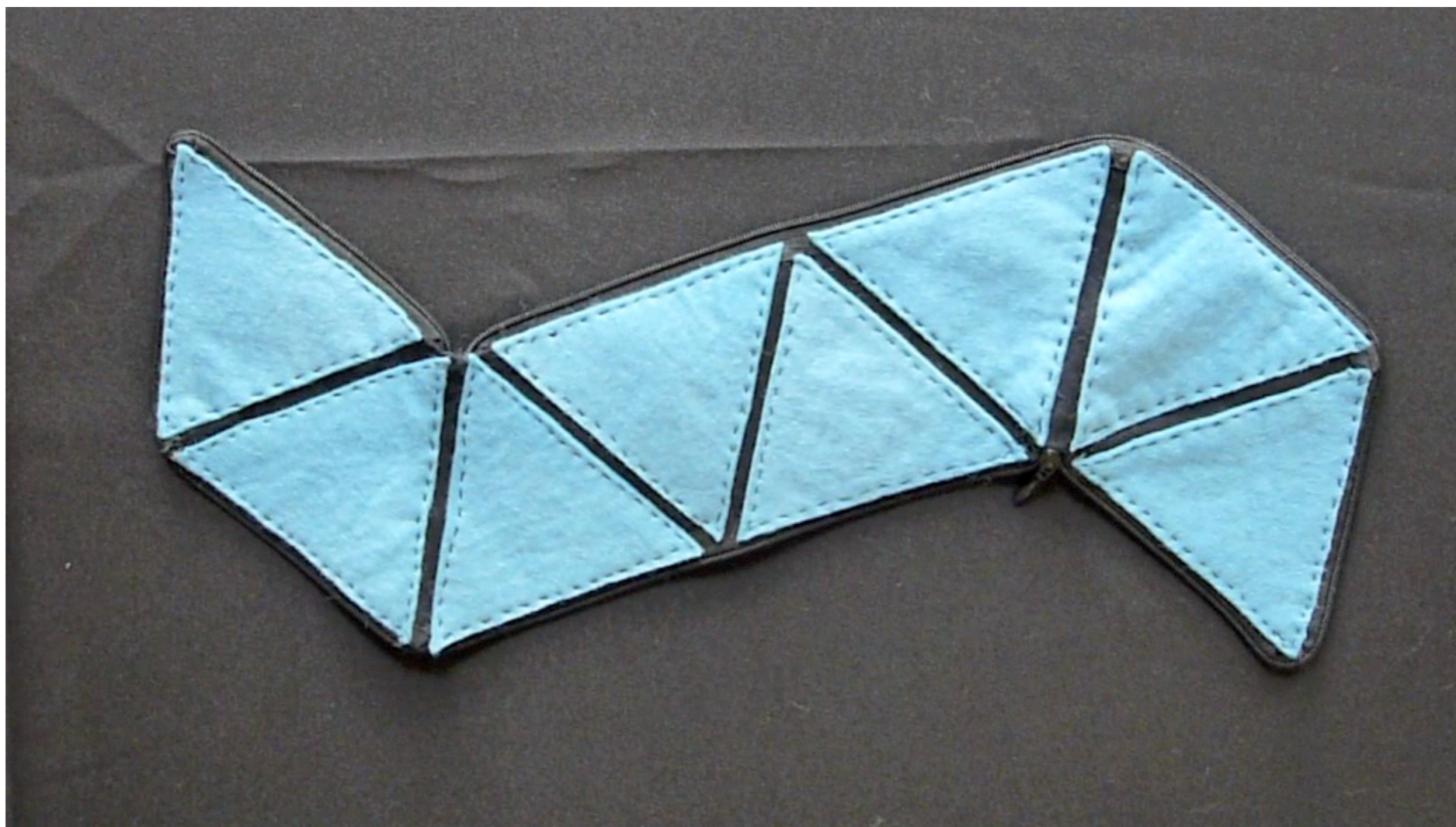
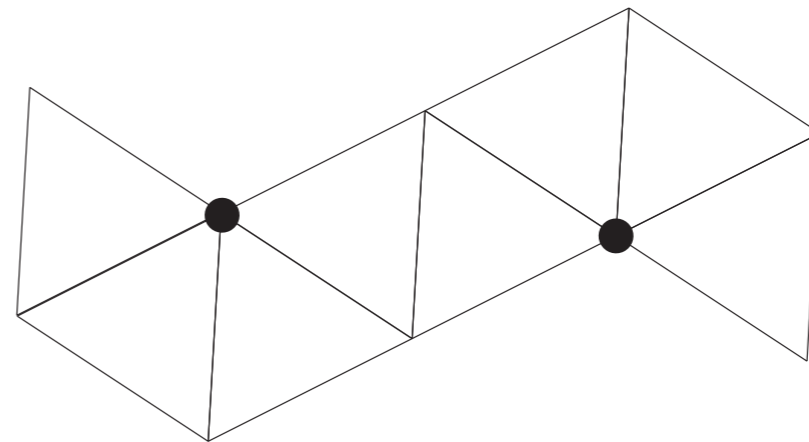
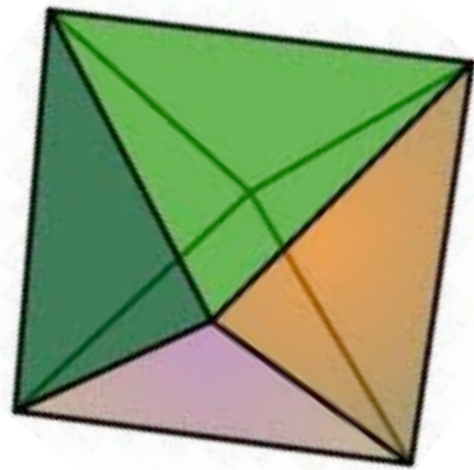
Unfolding Polyhedra—Octahedron

all unfoldings





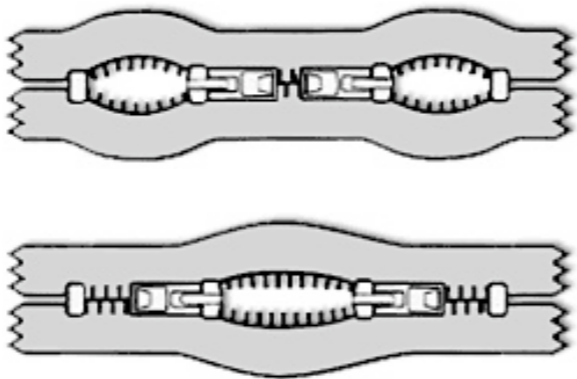
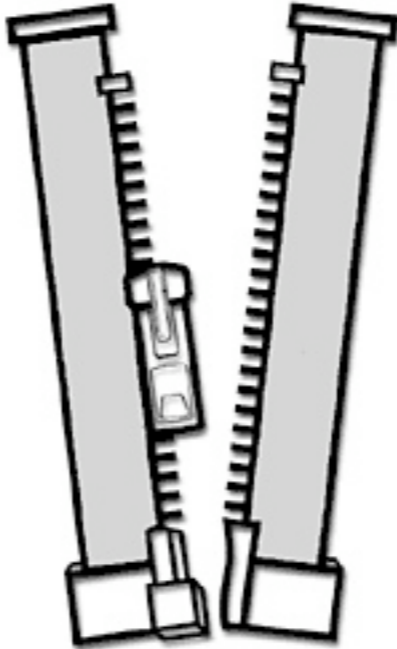
Zipper Unfoldings of Polyhedra—Octahedron



Zippers

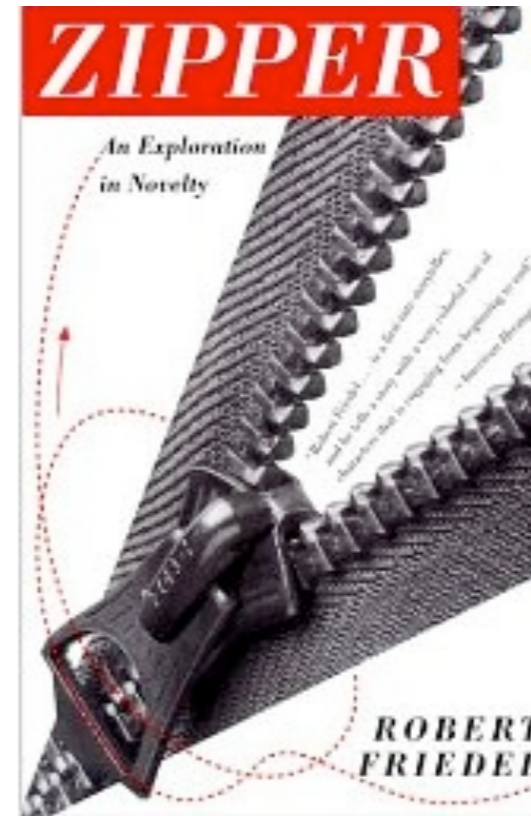
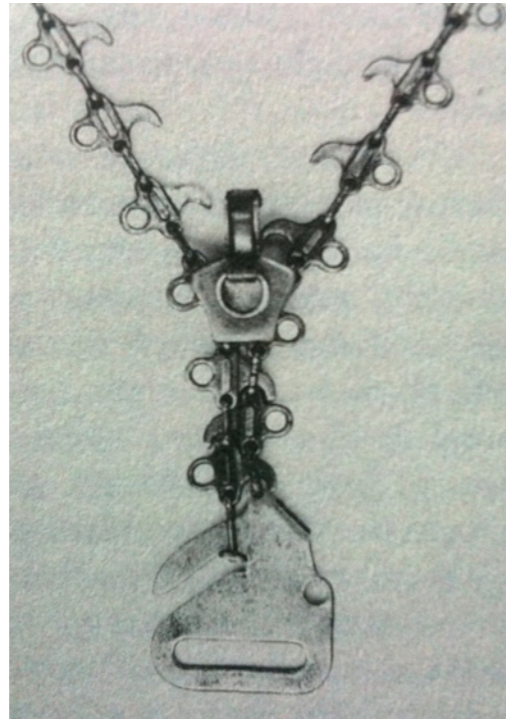


separating zipper



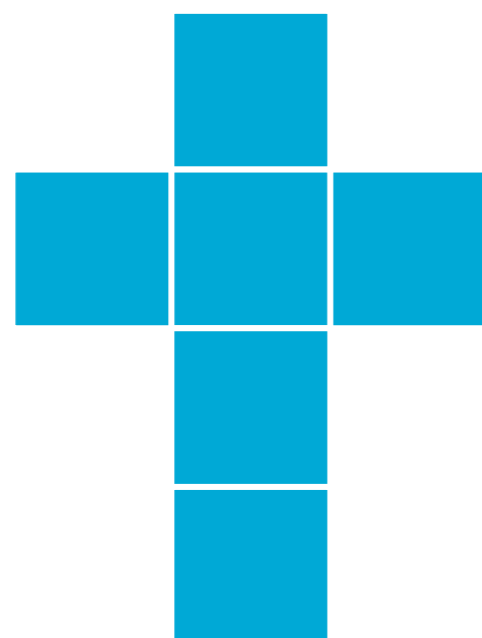
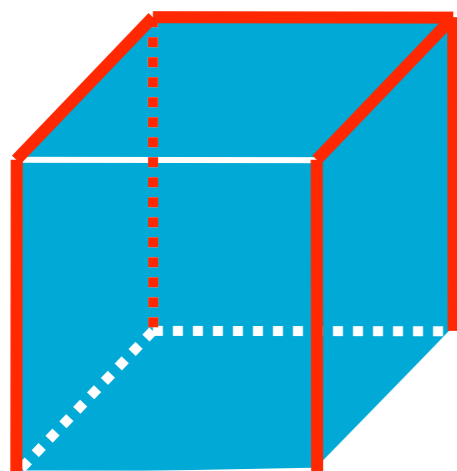
multiple toggles
(cosmetic)

Zippers

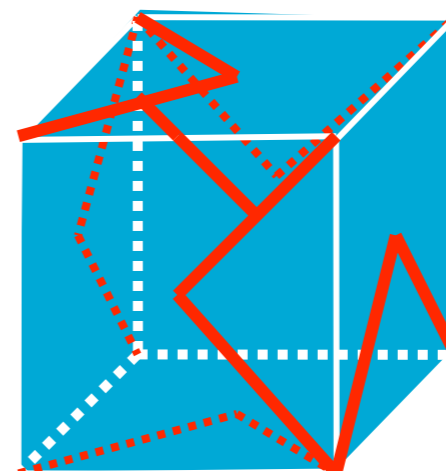


- 1891 patent by Whitcomb Judson
- novel, but not practical (“If skirt is to be washed, remove fastener.”)
- named “zipper” by B.F. Goodrich company in 1920’s
- ubiquitous but superfluous

Edge Cuts versus Face Cuts



edge cuts



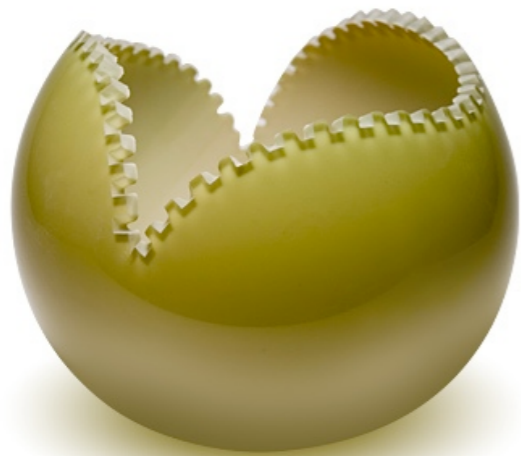
face cuts

Zipper edge cuts = *Hamiltonian unfolding* [Shepherd '75]

Zipper Edge Cuts (Hamiltonian Unfolding)

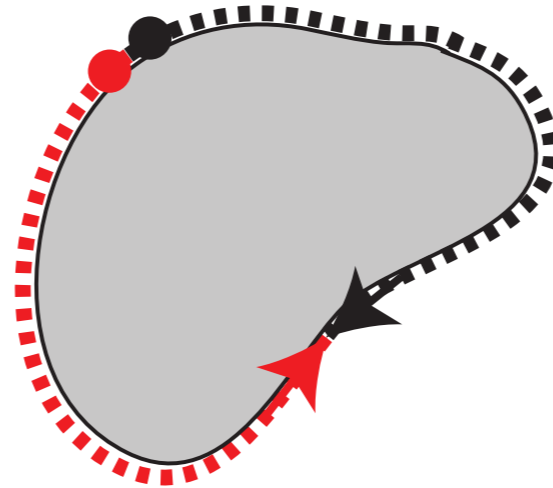
What is a zipper unfolding of a polyhedron?

on the polyhedron
the cut is a simple path

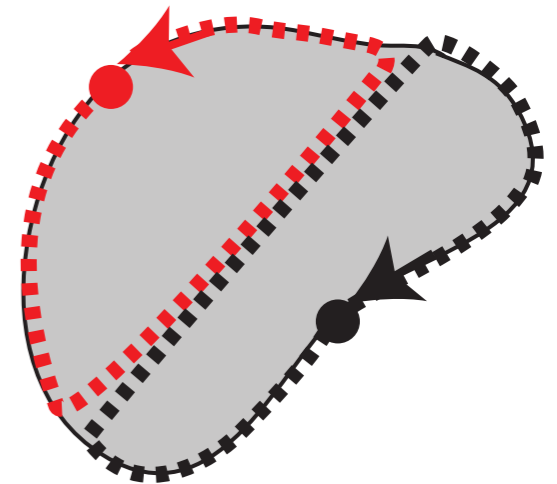


Nick Chase

on the polygon



this is forbidden



Outline of Talk

Convex Polyhedra

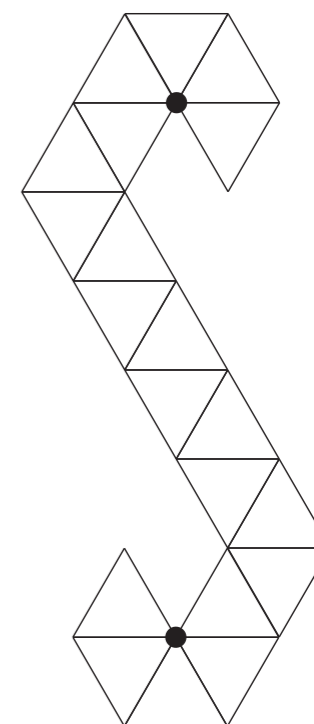
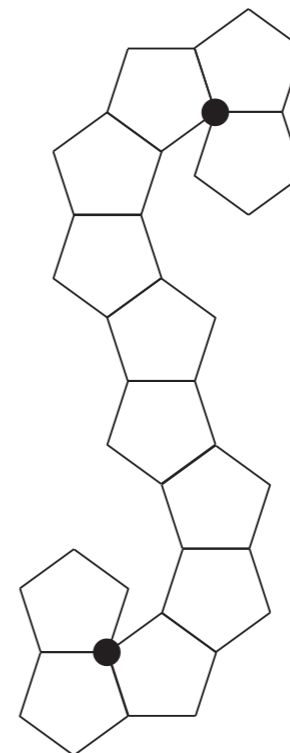
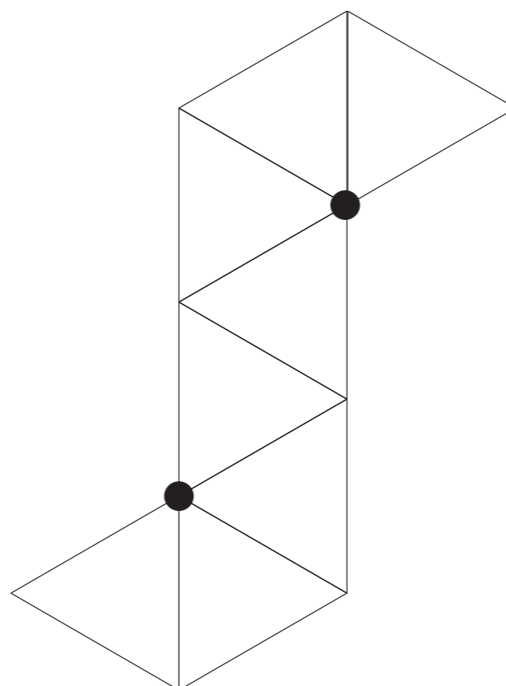
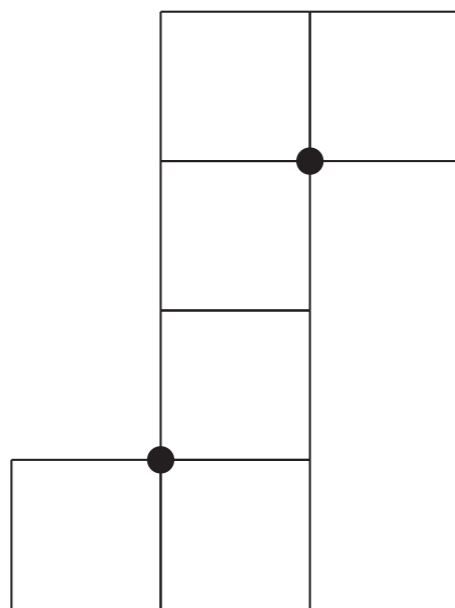
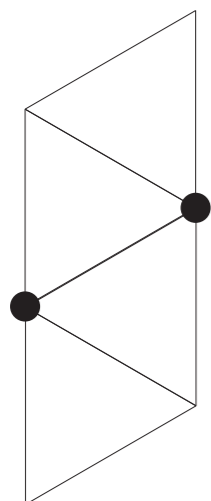
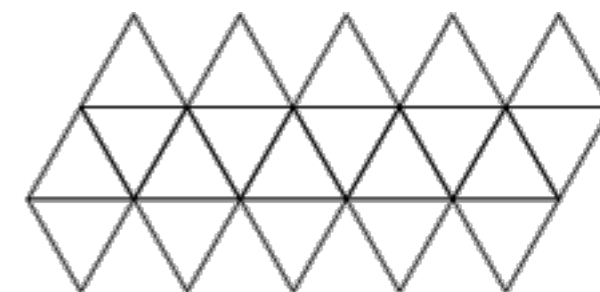
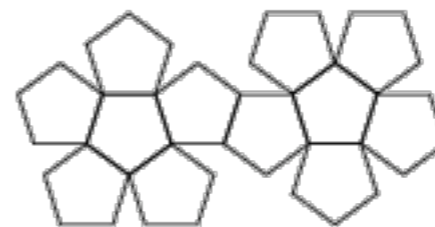
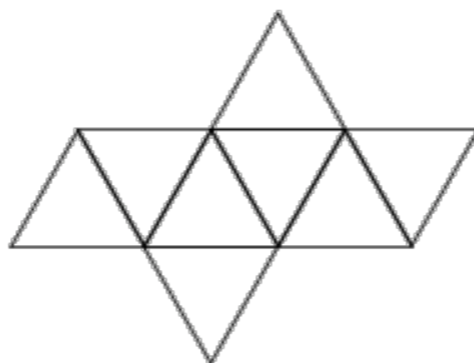
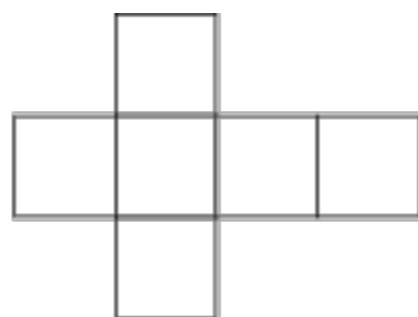
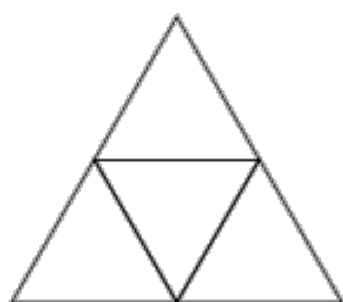
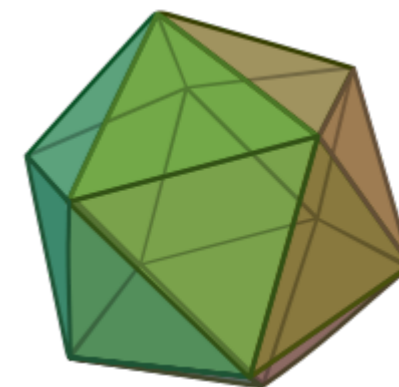
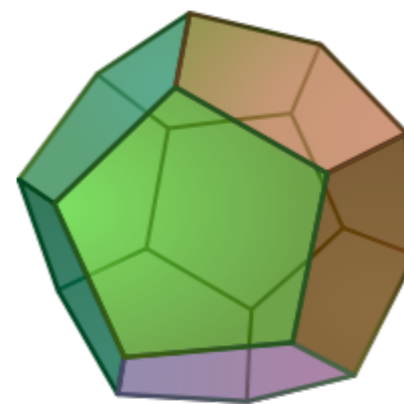
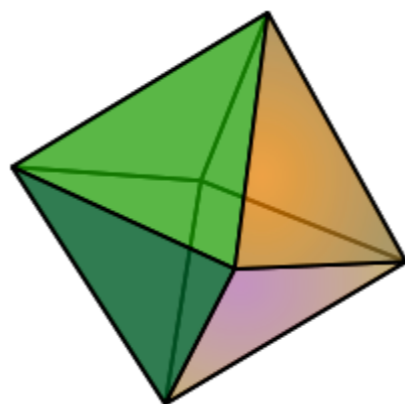
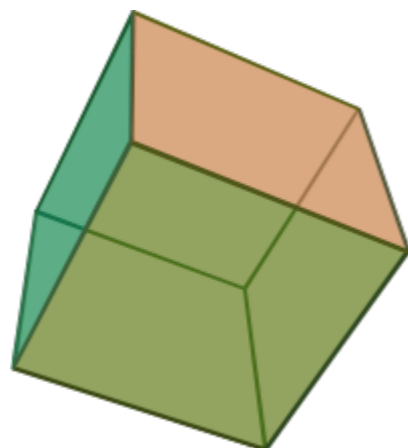
Platonic Solids

Archimedean Solids

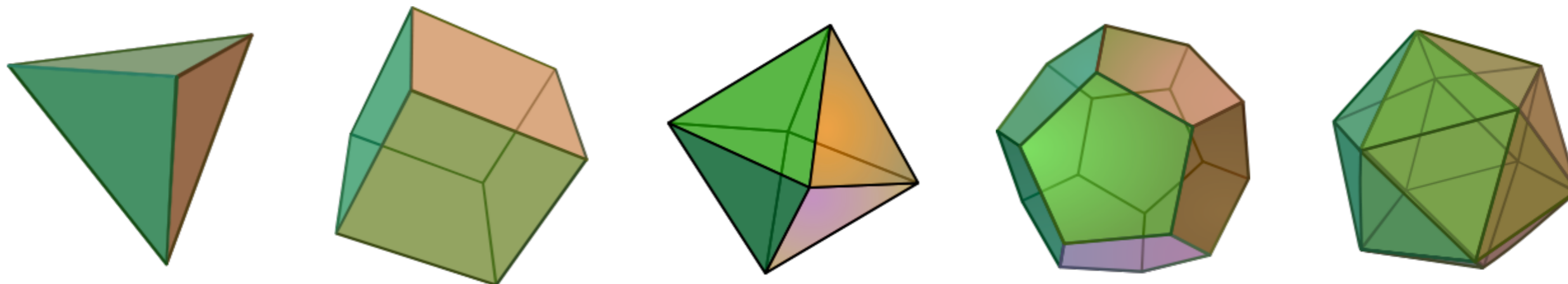
Polyhedral Manifolds

Polyhedral Complexes

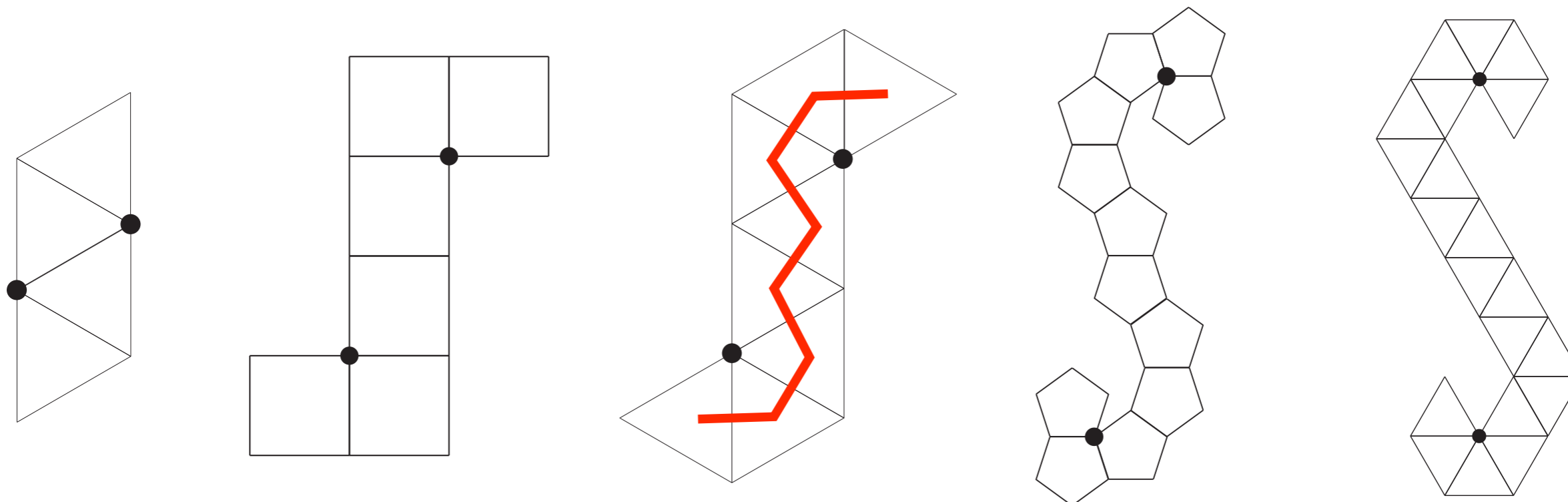
Platonic Solids



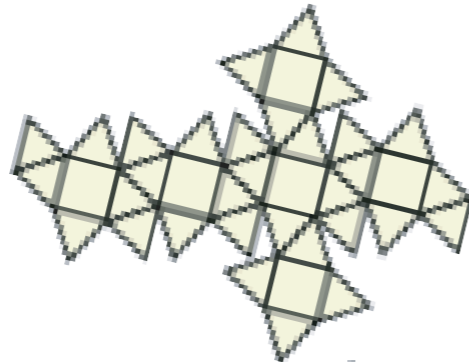
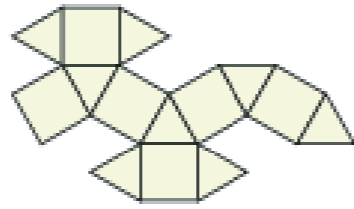
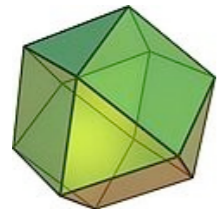
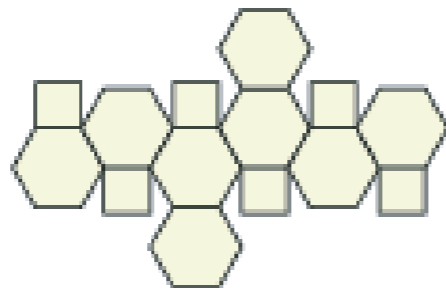
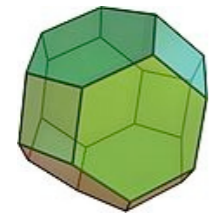
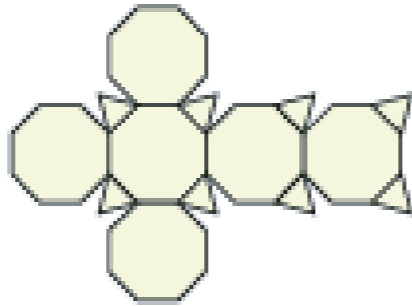
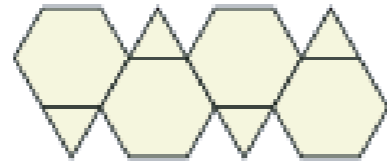
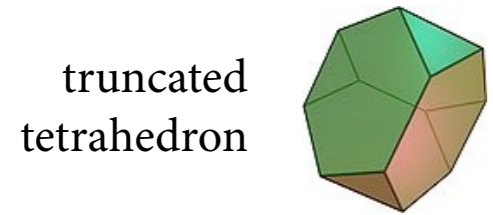
Platonic Solids



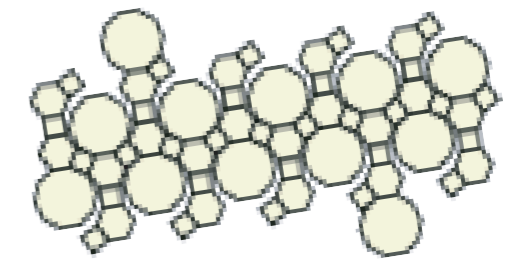
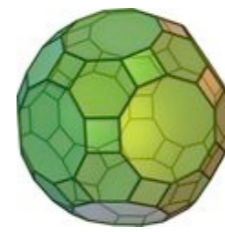
These are *doubly Hamiltonian*—the cut is a path and faces are joined in a path.



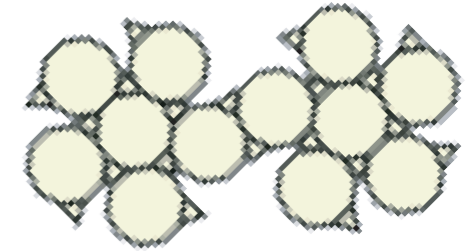
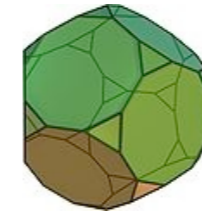
Archimedean Solids



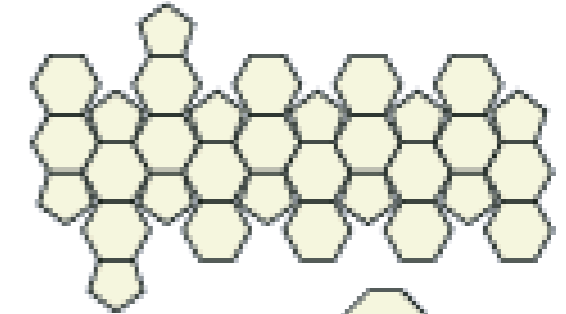
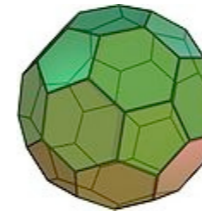
great rhombicosidodecahedron



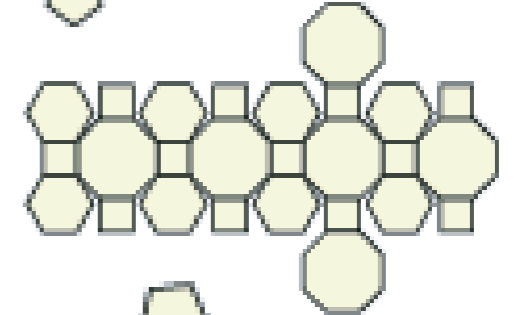
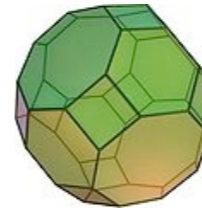
truncated dodecahedron



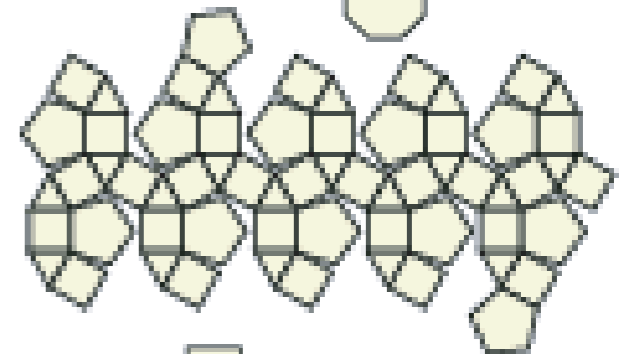
truncated icosahedron



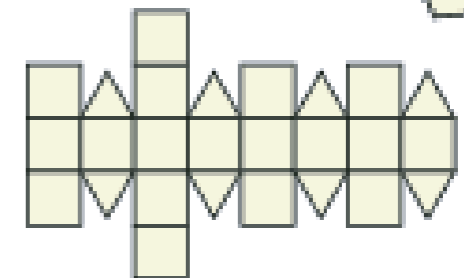
great rhombicuboctahedron



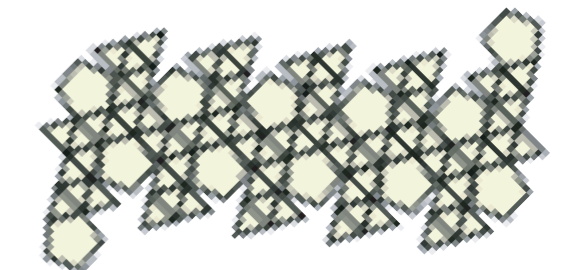
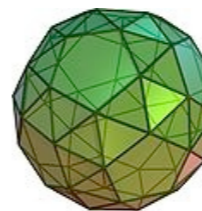
small rhombicosidodecahedron




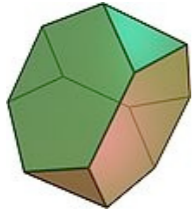
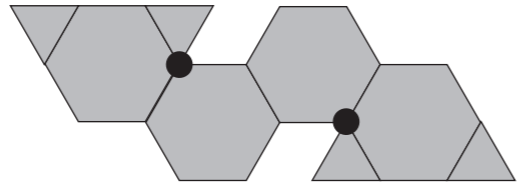

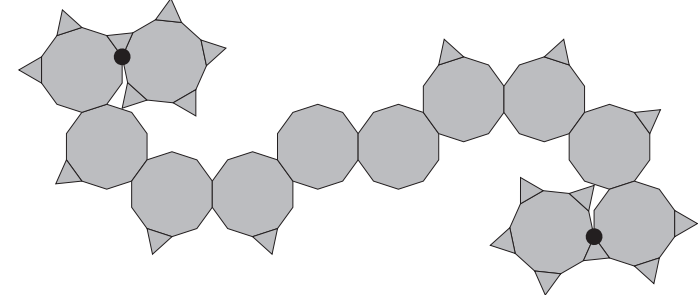

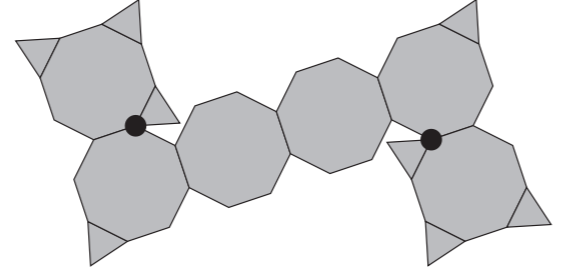

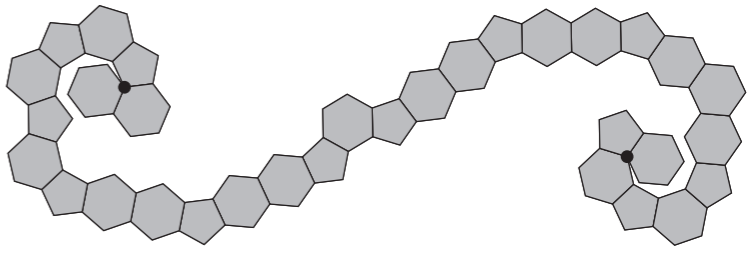

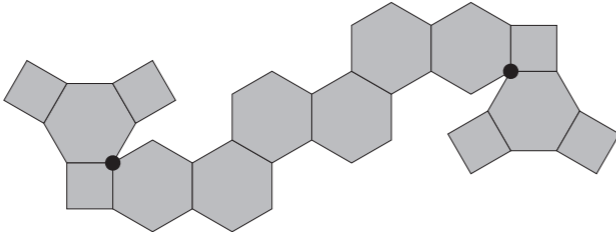
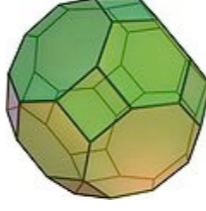
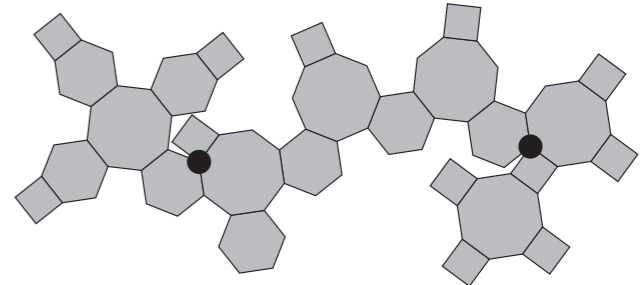
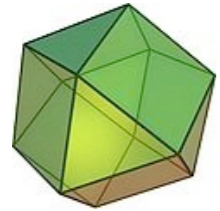
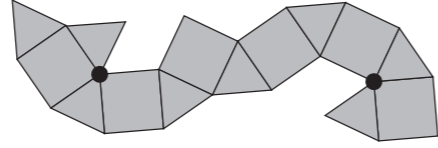

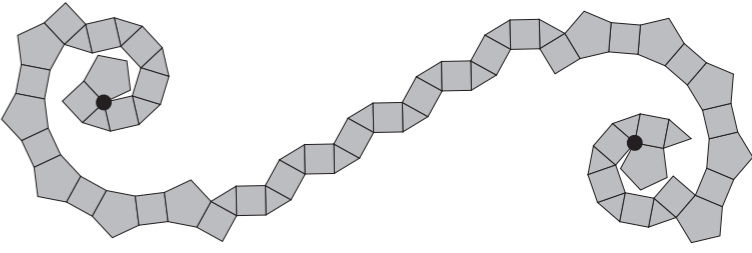

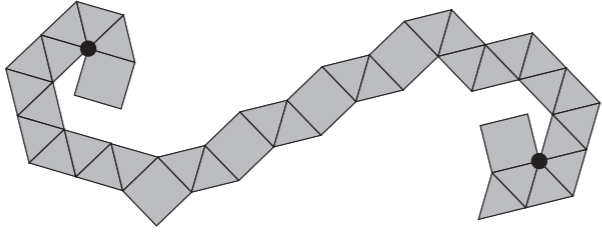

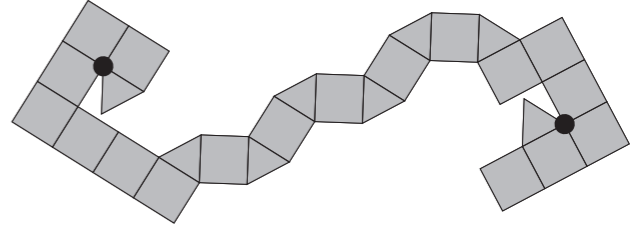

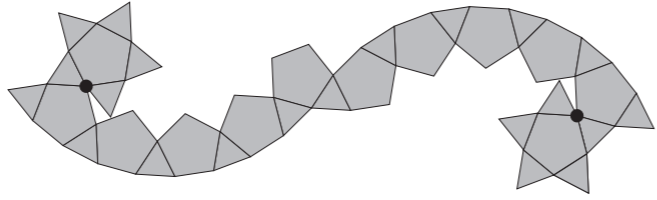

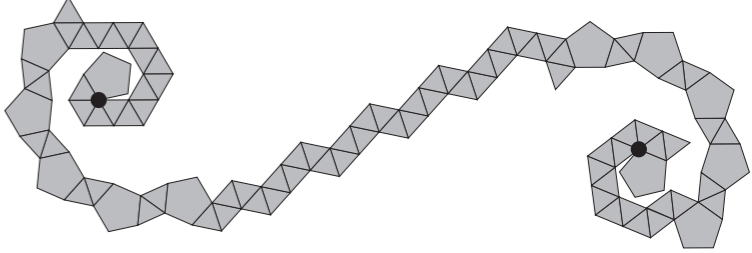
small rhombicuboctahedron



snub dodecahedron

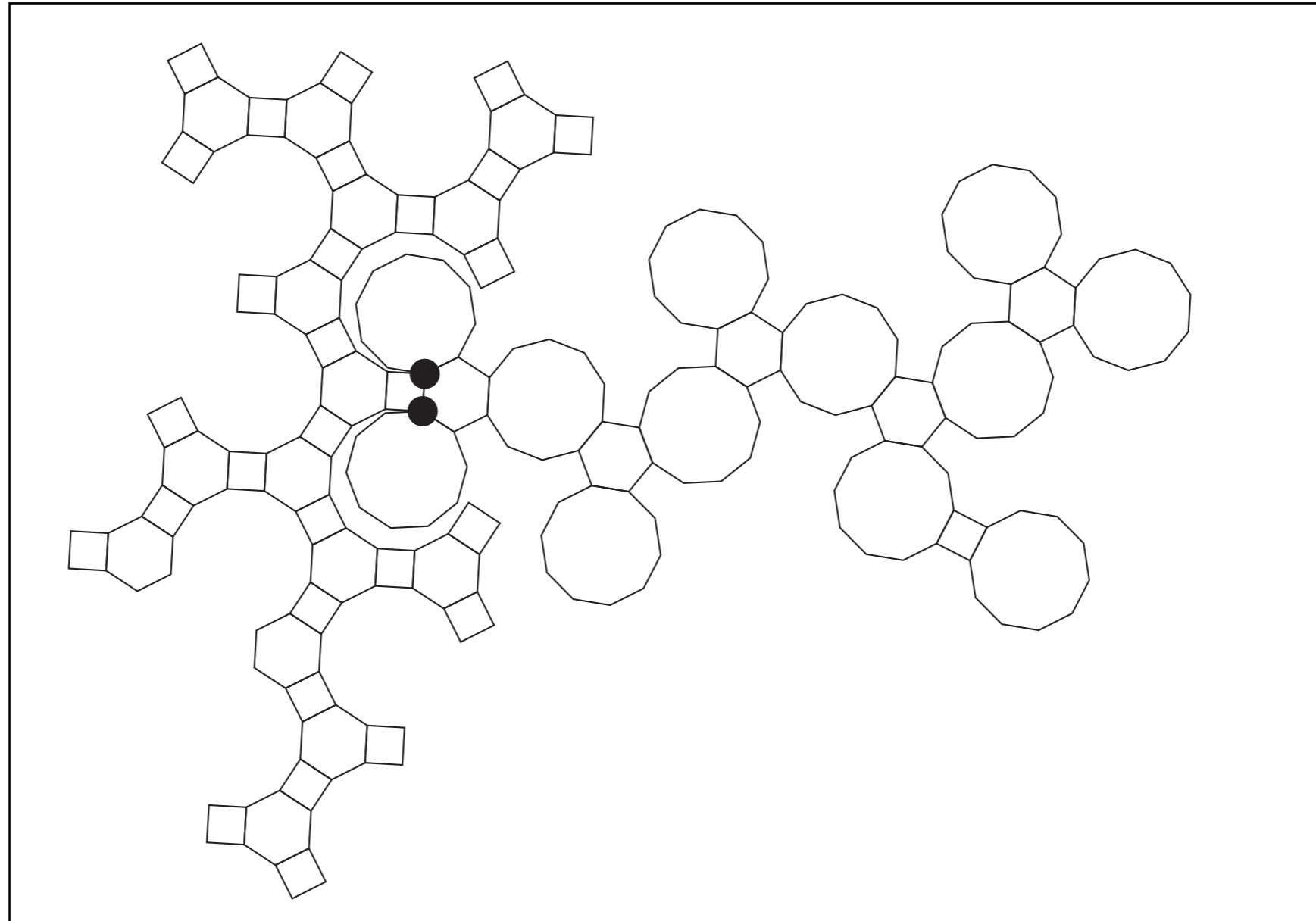
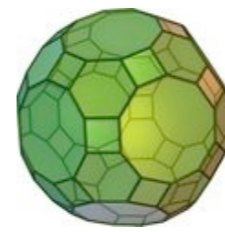


Archimedean Solids

			great rhombicosidodecahedron		
truncated tetrahedron			truncated dodecahedron		
truncated cube			truncated icosahedron		
truncated octahedron			great rhombicuboctahedron		
cuboctahedron			small rhombicosidodecahedron		
snub cube			small rhombicuboctahedron		
icosidodecahedron			snub dodecahedron		

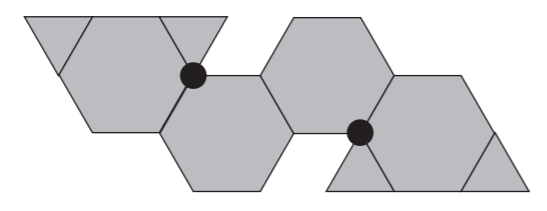
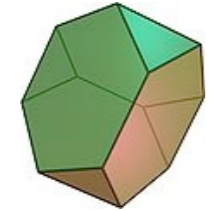
Archimedean Solids

great
rhombicosi-
dodecahedron

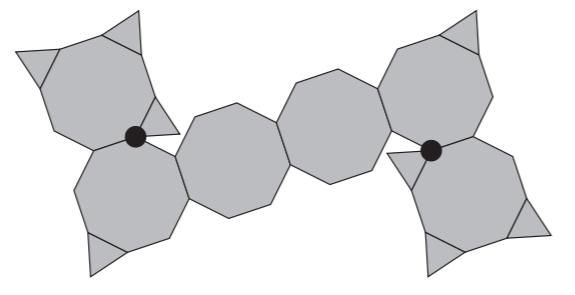


Archimedean Solids

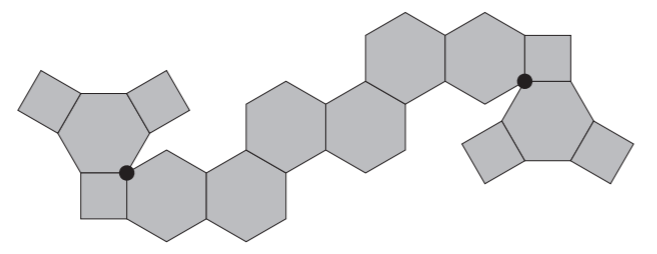
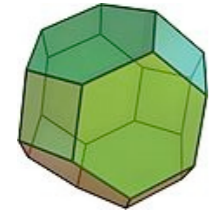
truncated tetrahedron



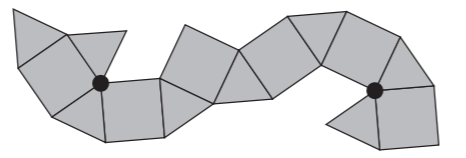
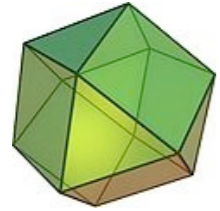
truncated cube



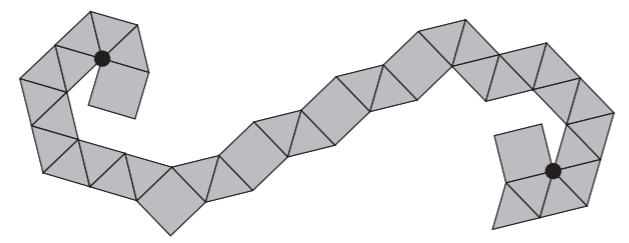
truncated octahedron



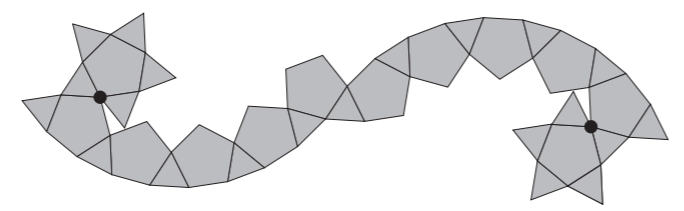
cuboctahedron



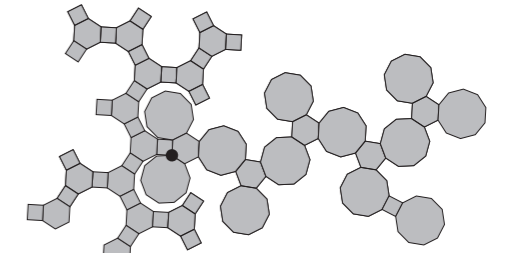
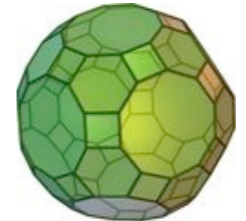
snub cube



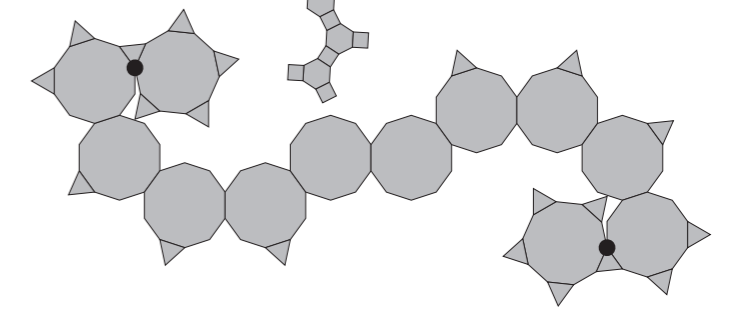
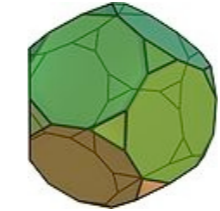
icosidodecahedron



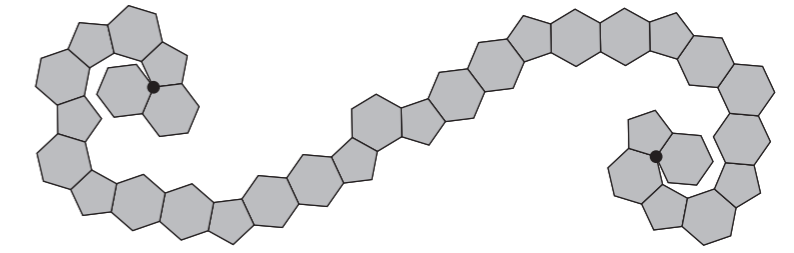
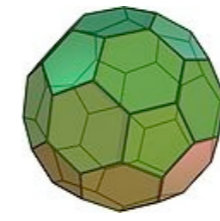
great rhombicosidodecahedron



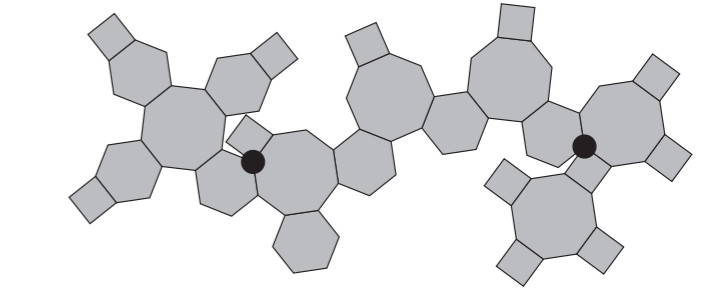
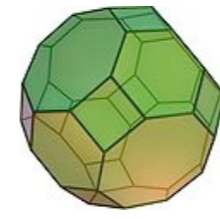
truncated dodecahedron



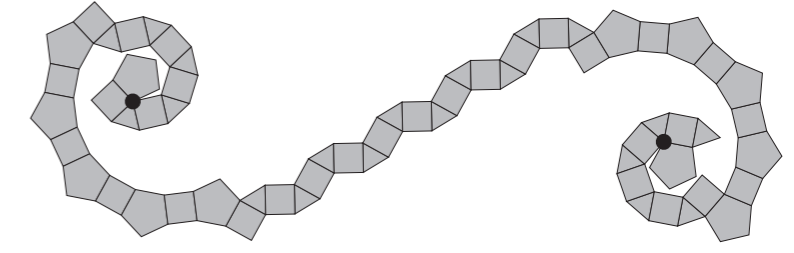
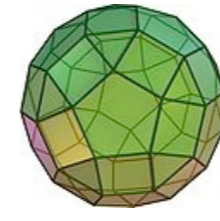
truncated icosahedron



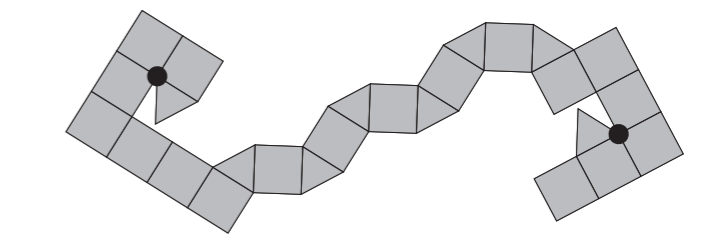
great rhombicuboctahedron



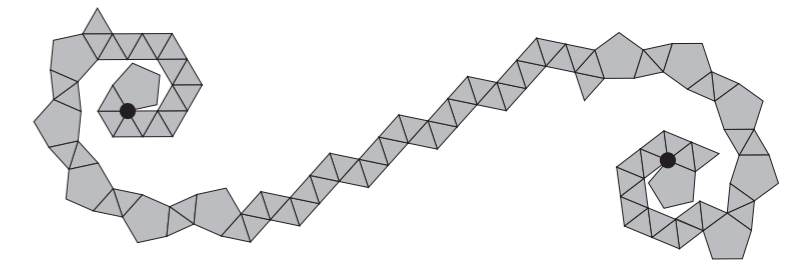
small rhombicosidodecahedron



small rhombicuboctahedron



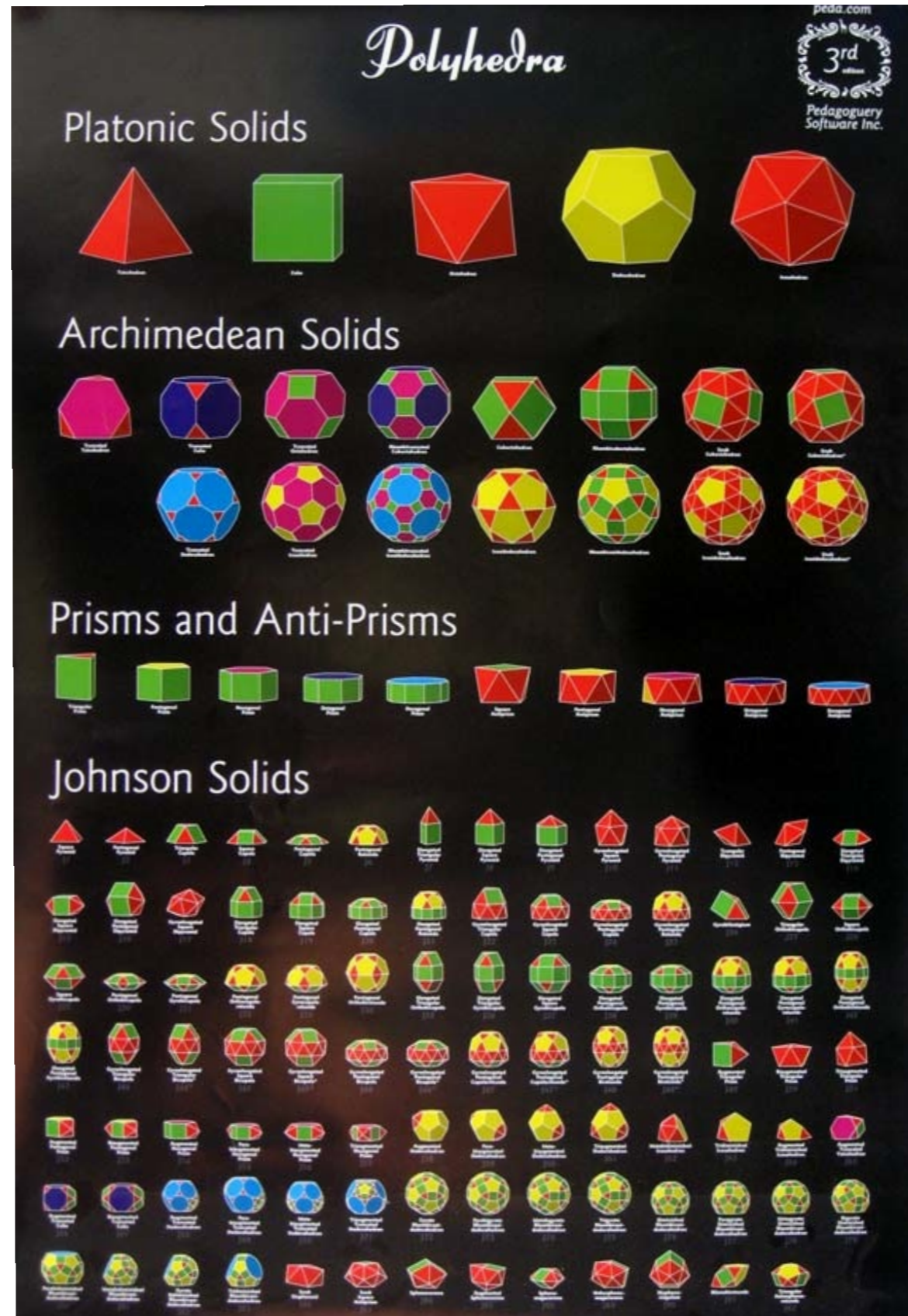
snub dodecahedron



What next?

these have
zipper unfoldings

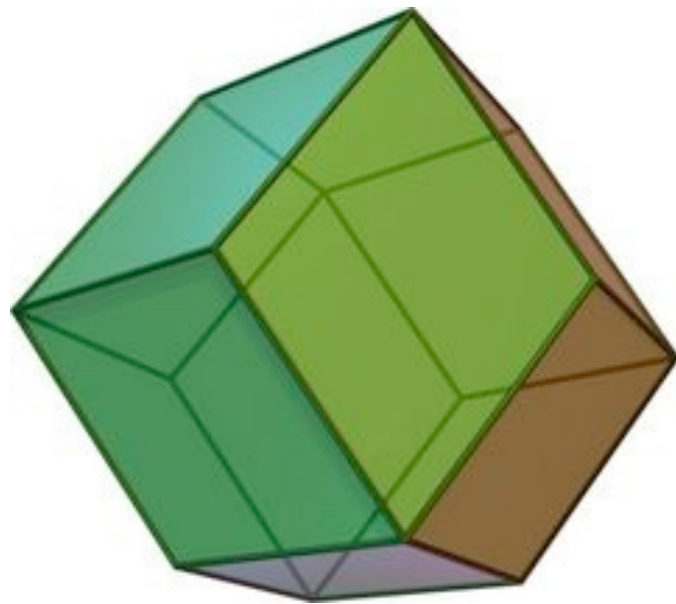
??



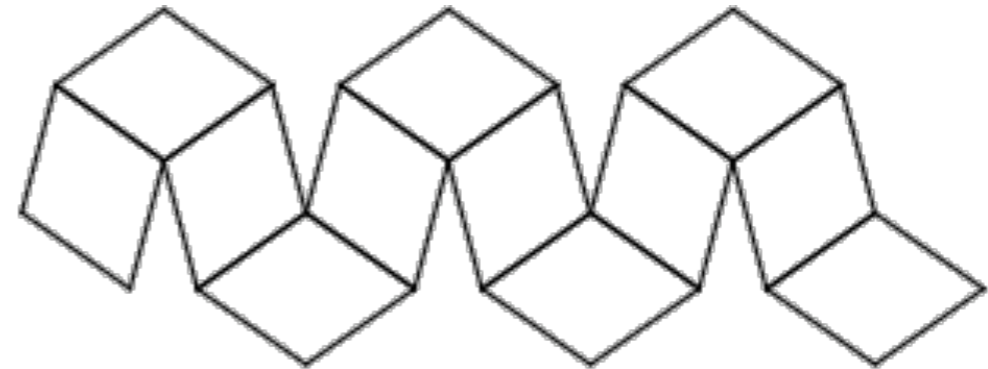
Peda Software Polyhedron Poster

Not all convex polyhedra have Hamiltonian unfoldings

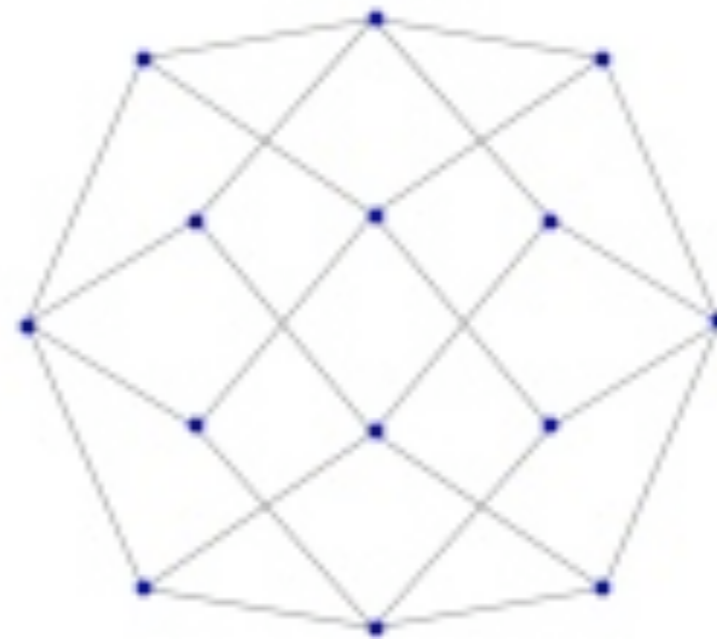
rhombic dodecahedron



net



its graph has no
Hamiltonian path



OPEN: find a convex polyhedron with no Hamiltonian unfolding but whose graph has a Hamiltonian path.

Unfolding Convex Polyhedra

unfolding

zipper unfolding

face
cuts

YES

Every convex polyhedron
has an unfolding—
star, source unfolding.

OPEN

Does every convex polyhedron
have a zipper unfolding?

edge
cuts

OPEN

Does every convex polyhedron
have an edge unfolding?

NO

Not every convex polyhedron
has a Hamiltonian unfolding.

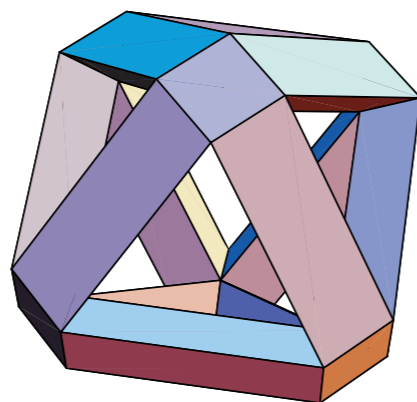
Polyhedral Manifolds

polyhedral manifold—a finite union of planar polygons in 3D s.t. every point has a neighbourhood homeomorphic to a disk

may be non-convex



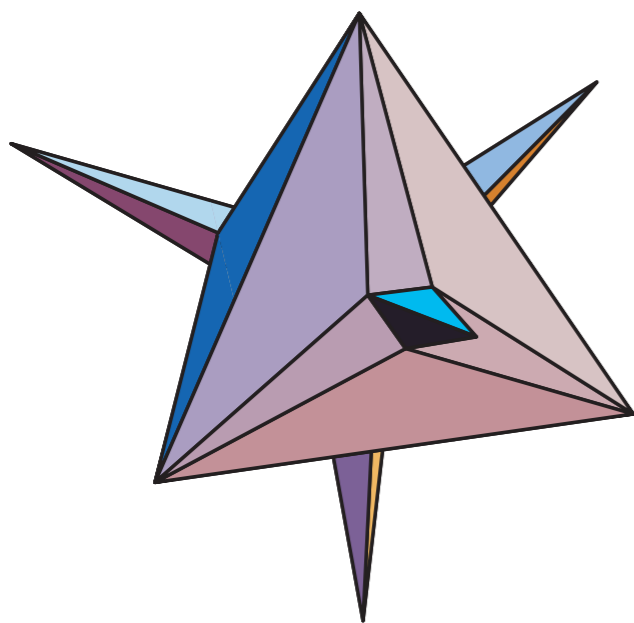
may have genus $\neq 0$



Magnus Wenninger

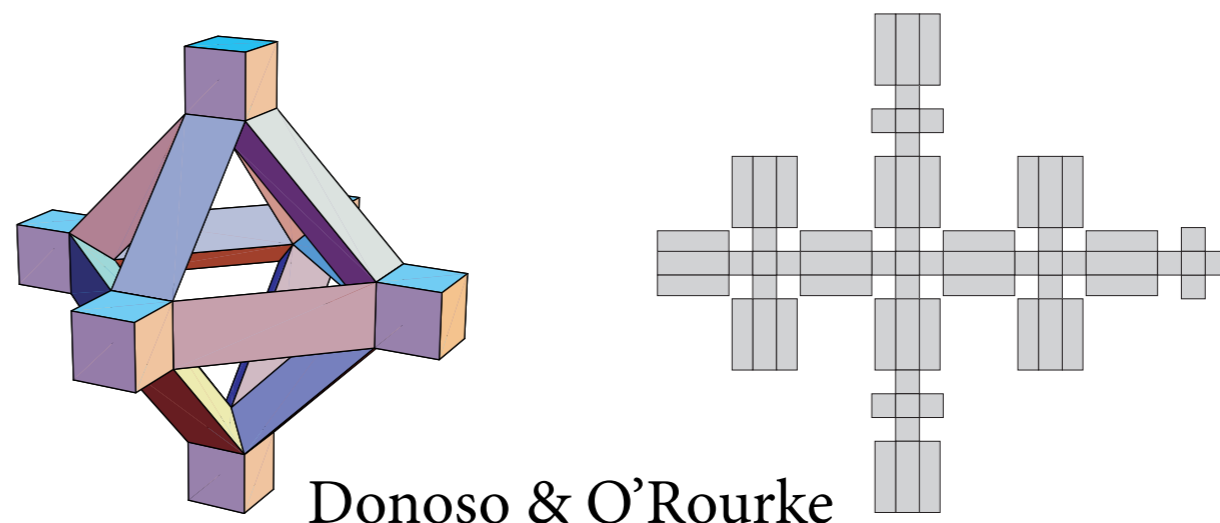
Polyhedral Manifolds—Unfolding

a “nice” non-convex polyhedron with no edge unfolding



Bern, Demaine, Eppstein, Kuo, Mantler, Snoeyink, 2003.

some higher genus polyhedral manifolds with edge unfoldings



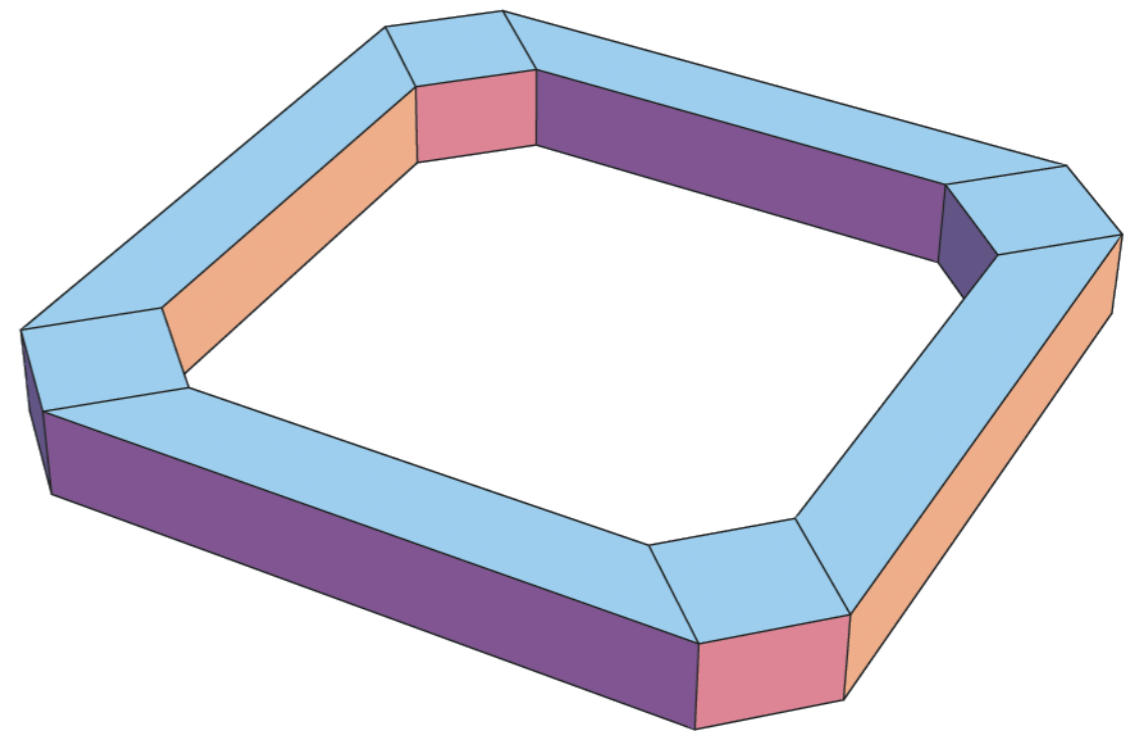
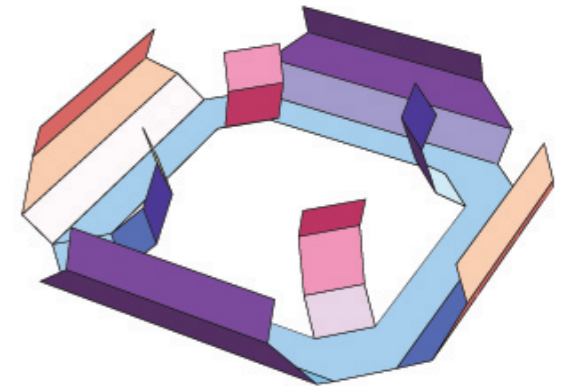
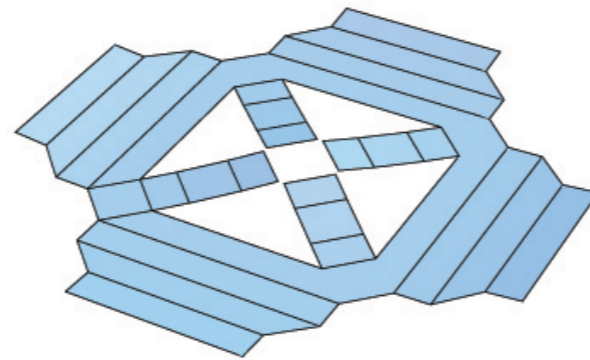
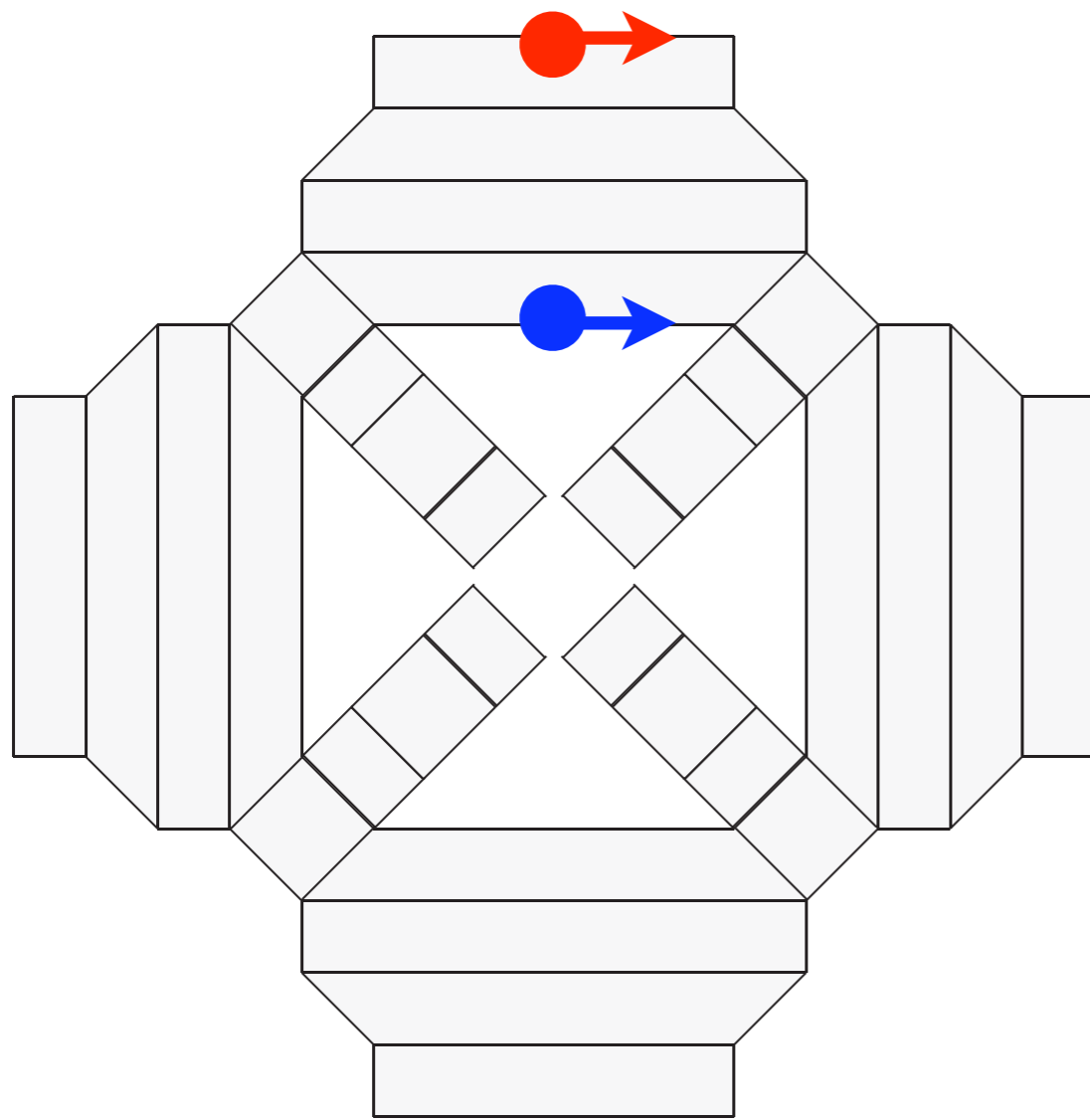
Donoso & O'Rourke



Császár torus (net by Lutz, picture by Polthier)

OPEN: Does every polyhedral manifold have a [general] unfolding?

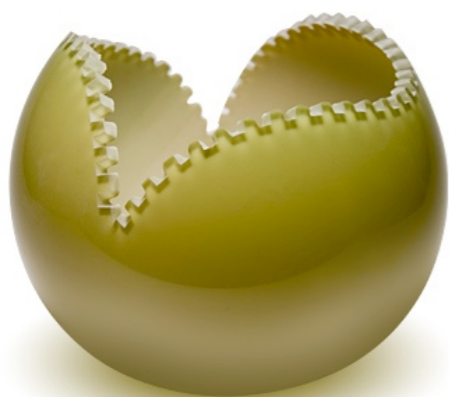
Polyhedral Manifolds—Zipper Unfoldings



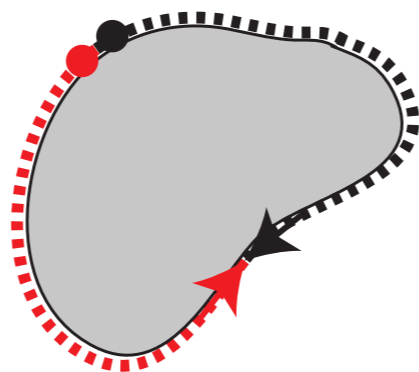
use separating zipper

Polyhedral Manifolds—Zipper Unfoldings

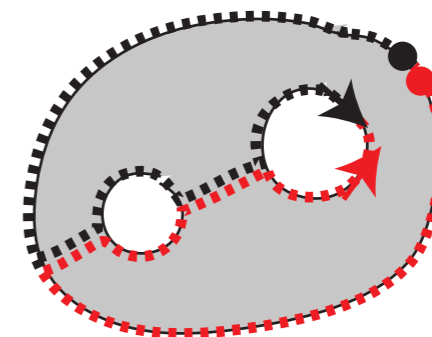
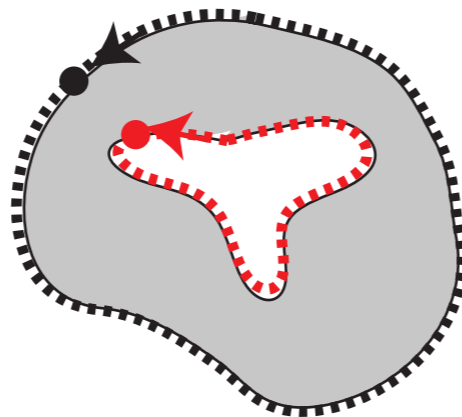
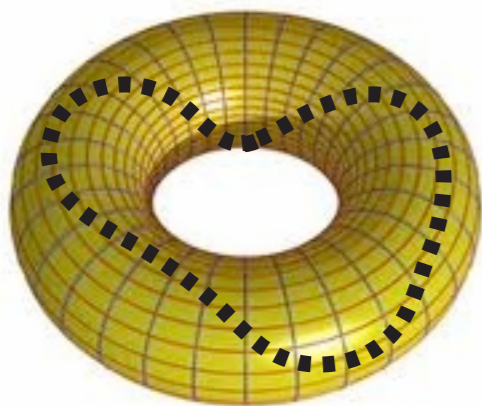
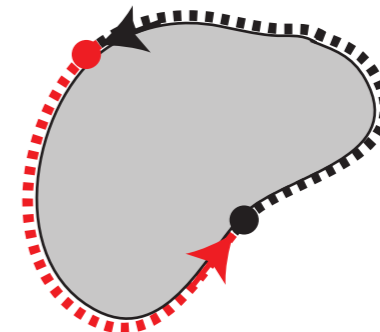
Theorem. If P is a polyhedral manifold that has a zipper unfolding to a planar polygonal region F , then either P is a polyhedron and F is a polygon, or—in the case of a separating zipper— P is a torus polyhedron and F is an annulus (with outer perimeter = inner perimeter).



Nick Chase

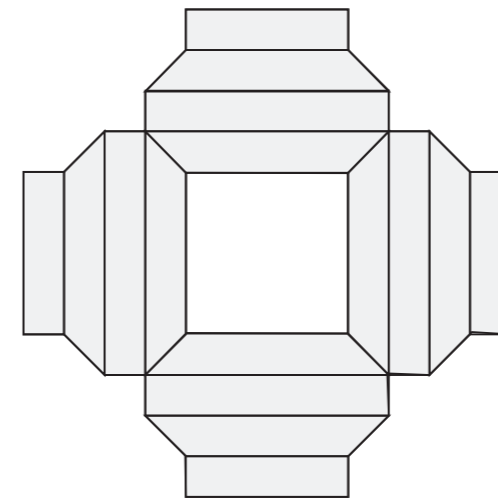
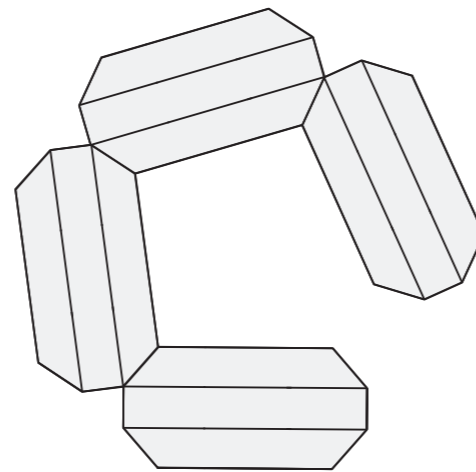
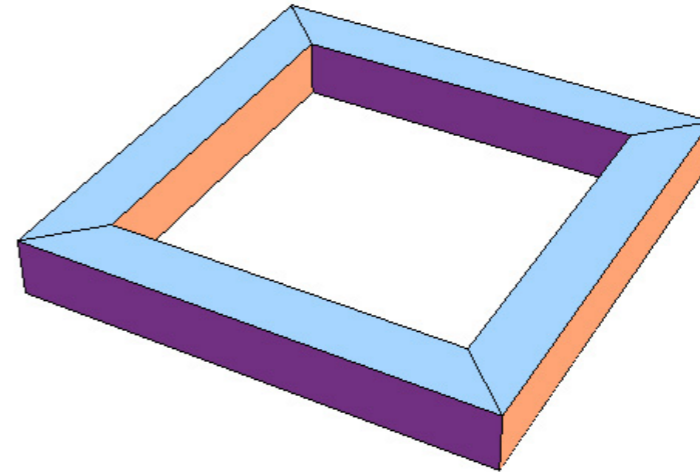
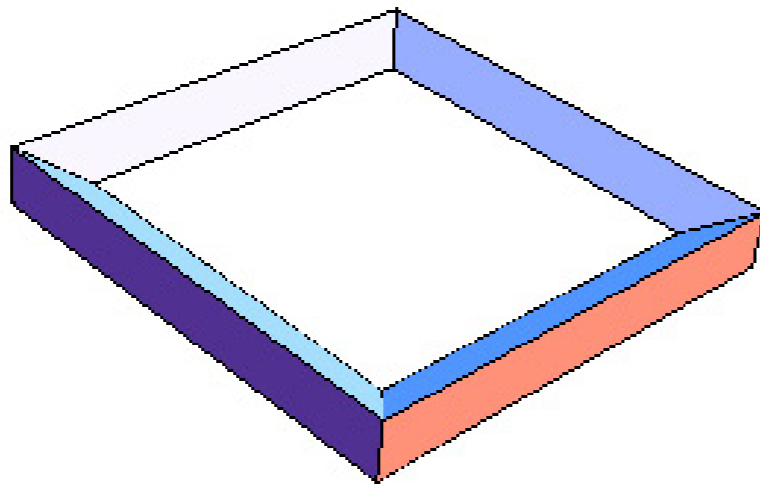


not possible



Polyhedral Manifolds—Zipper Unfoldings

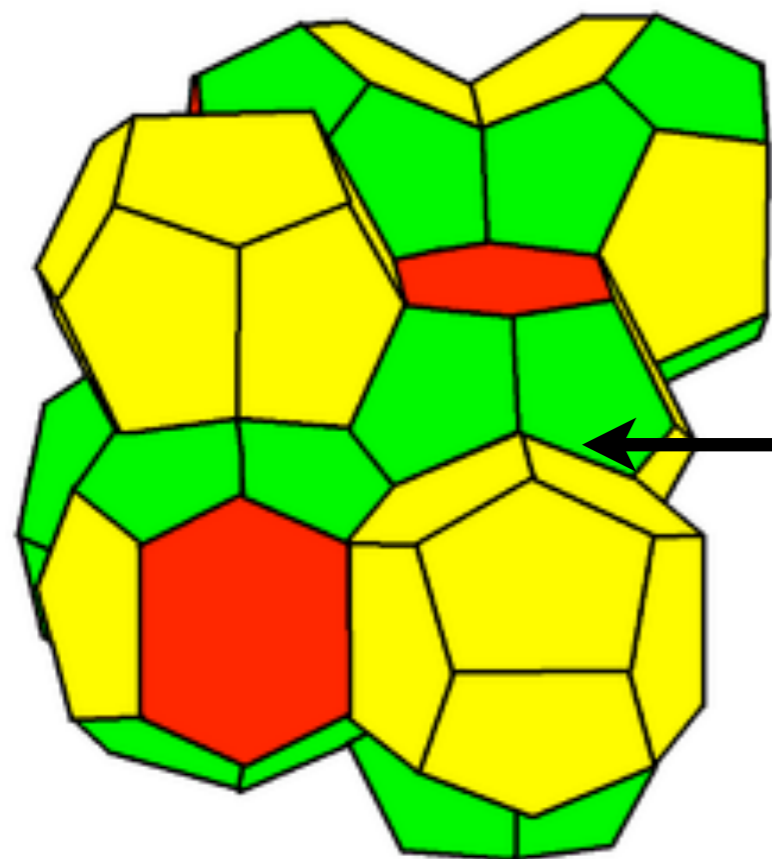
These have no zipper edge unfoldings.



Lemma. A zipper edge unfolding of a torus polyhedron is an annulus with faces forming a cycle with trees attached.

Polyhedral Complexes

polyhedral complex—a finite union of planar polygons in 3D s.t. intersections are edge-to-edge joins



Weaire-Phelan structure

only one face
at an edge

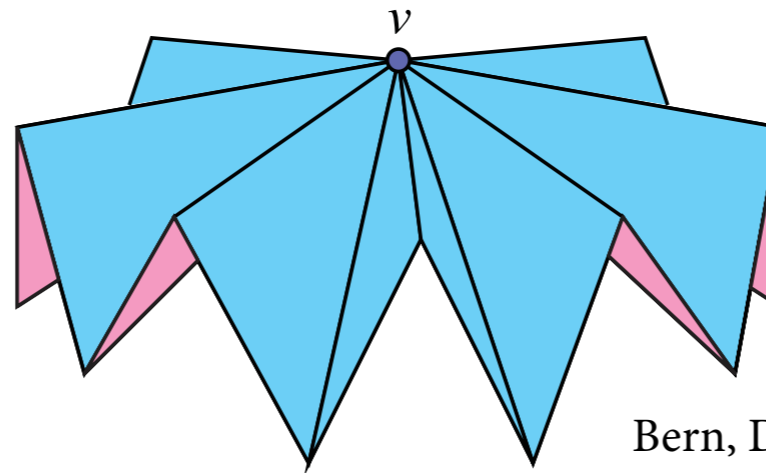


3 faces meeting
at an edge

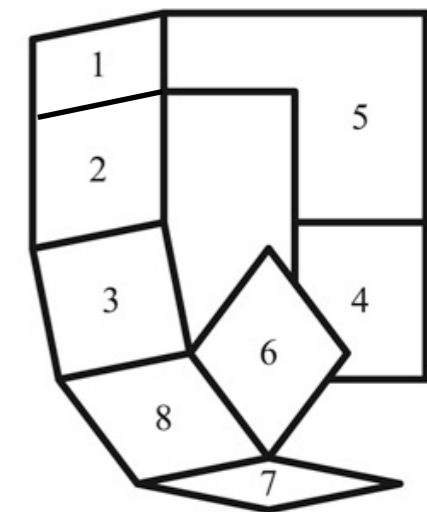
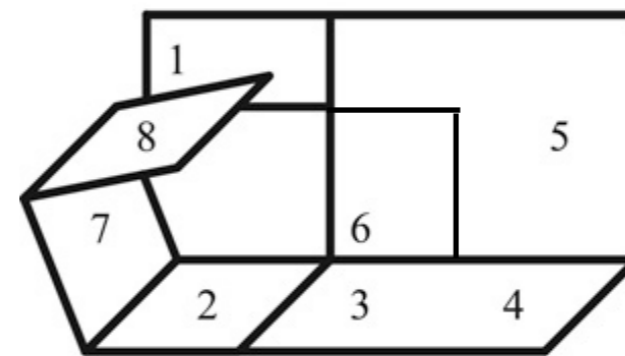
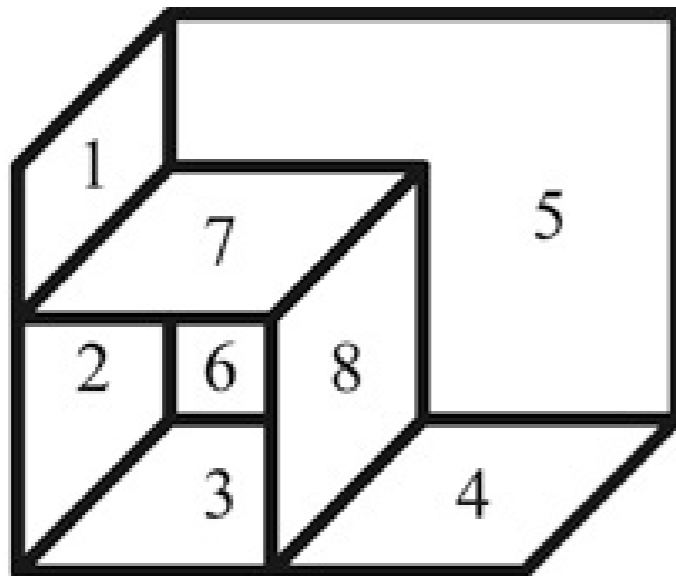


Polyhedral Complexes—Unfolding

cannot be
unfolded



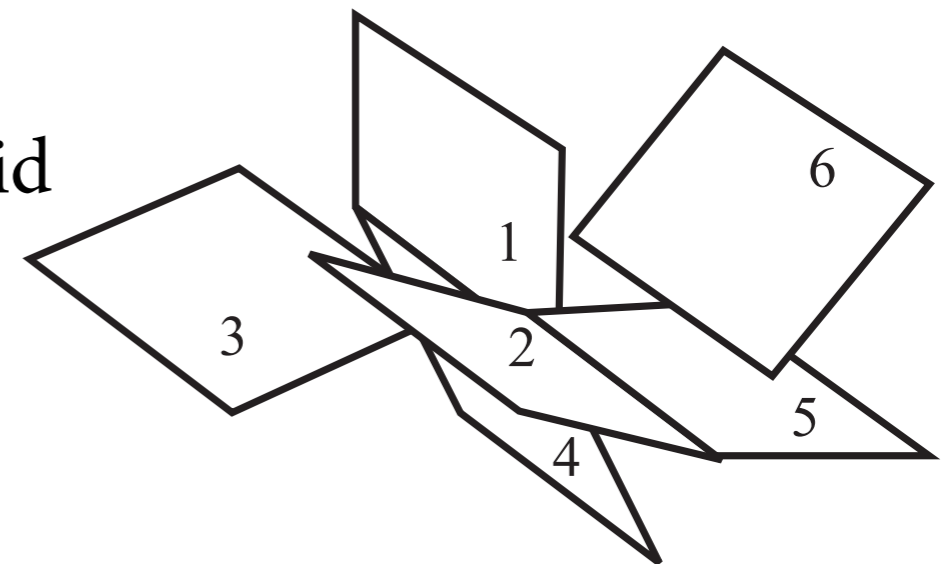
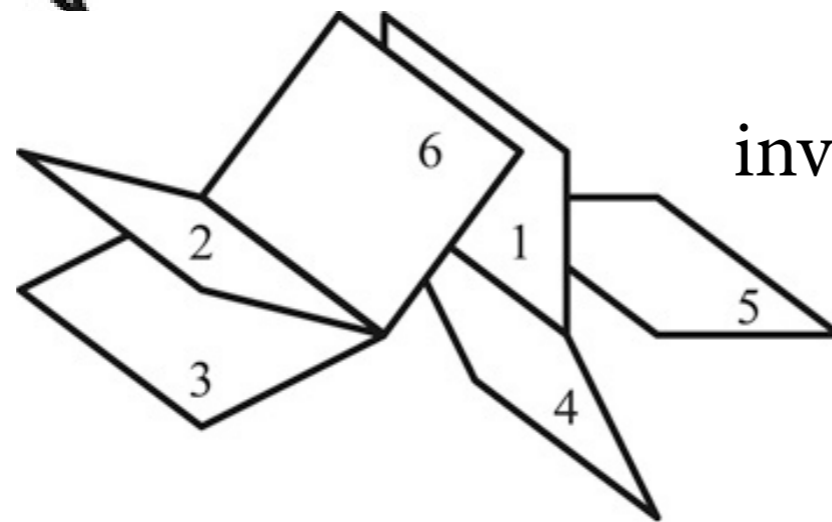
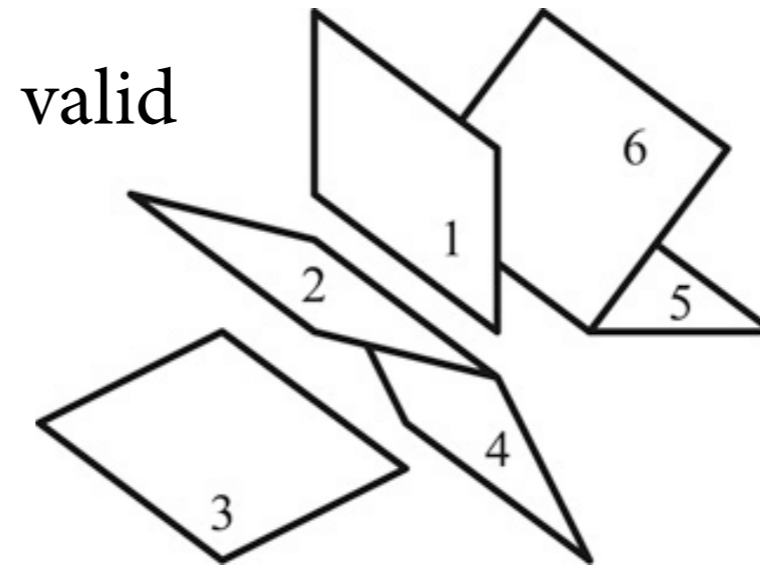
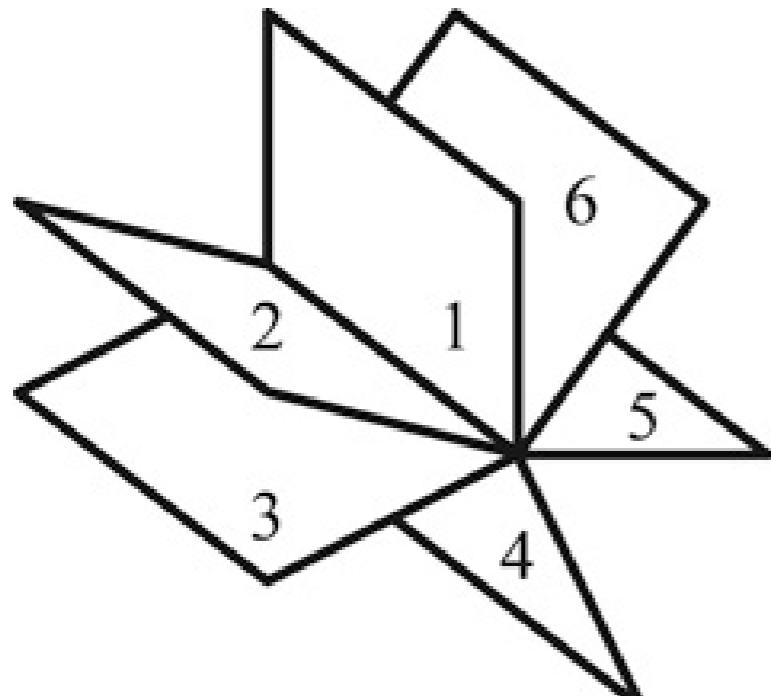
Bern, Demaine, Eppstein,
Kuo, Mantler, Snoeyink, 2003.



Liu & Tai, in *Computer-Aided Design*, 2007

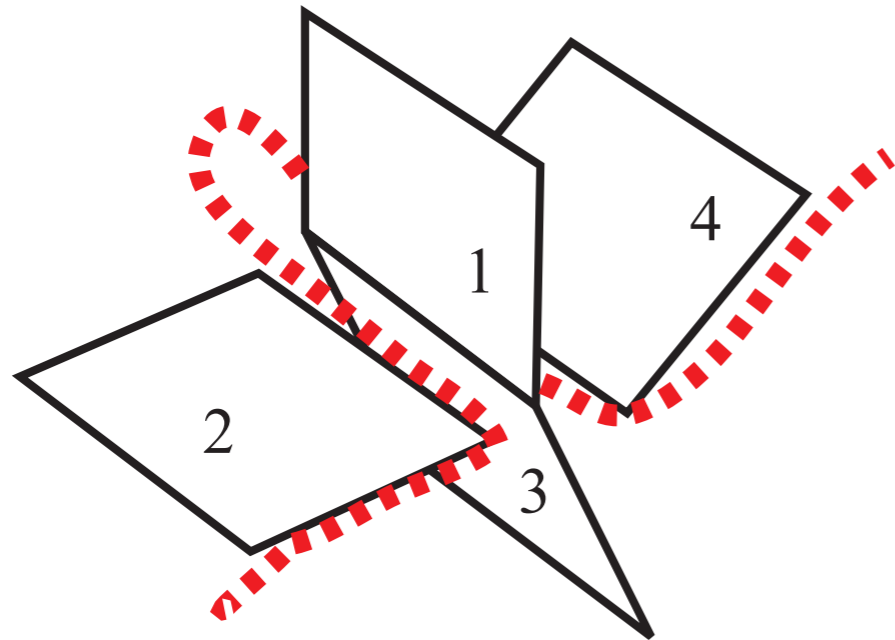
Polyhedral Complexes—Unfolding

What does unfolding mean when multiple faces meet at an edge?

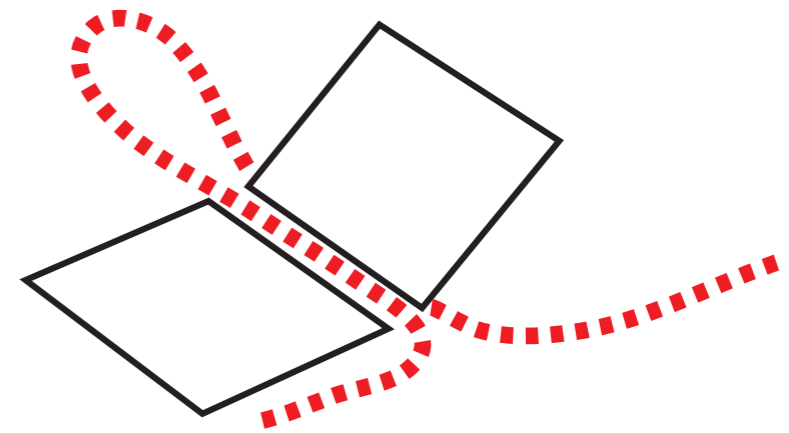


Liu & Tai, in *Computer-Aided Design*, 2007

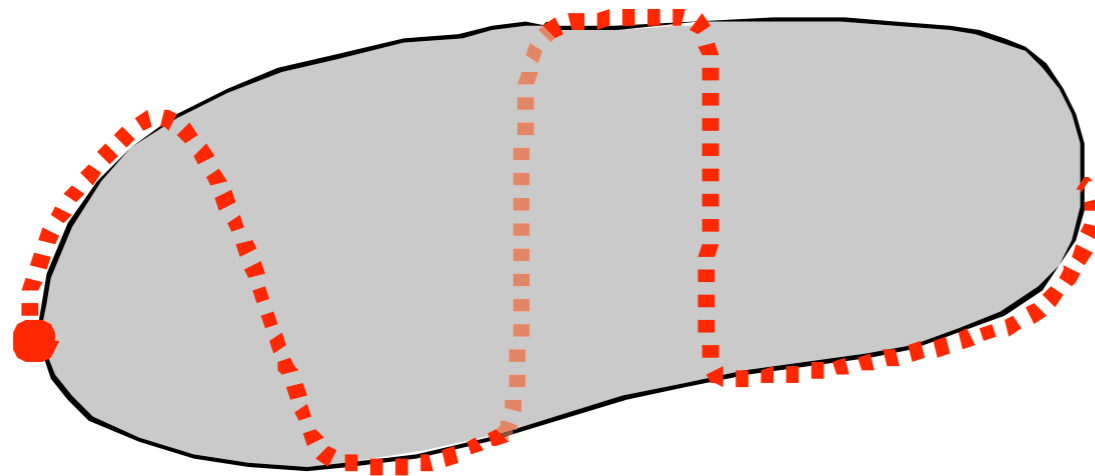
Polyhedral Complexes—Zipper Unfolding



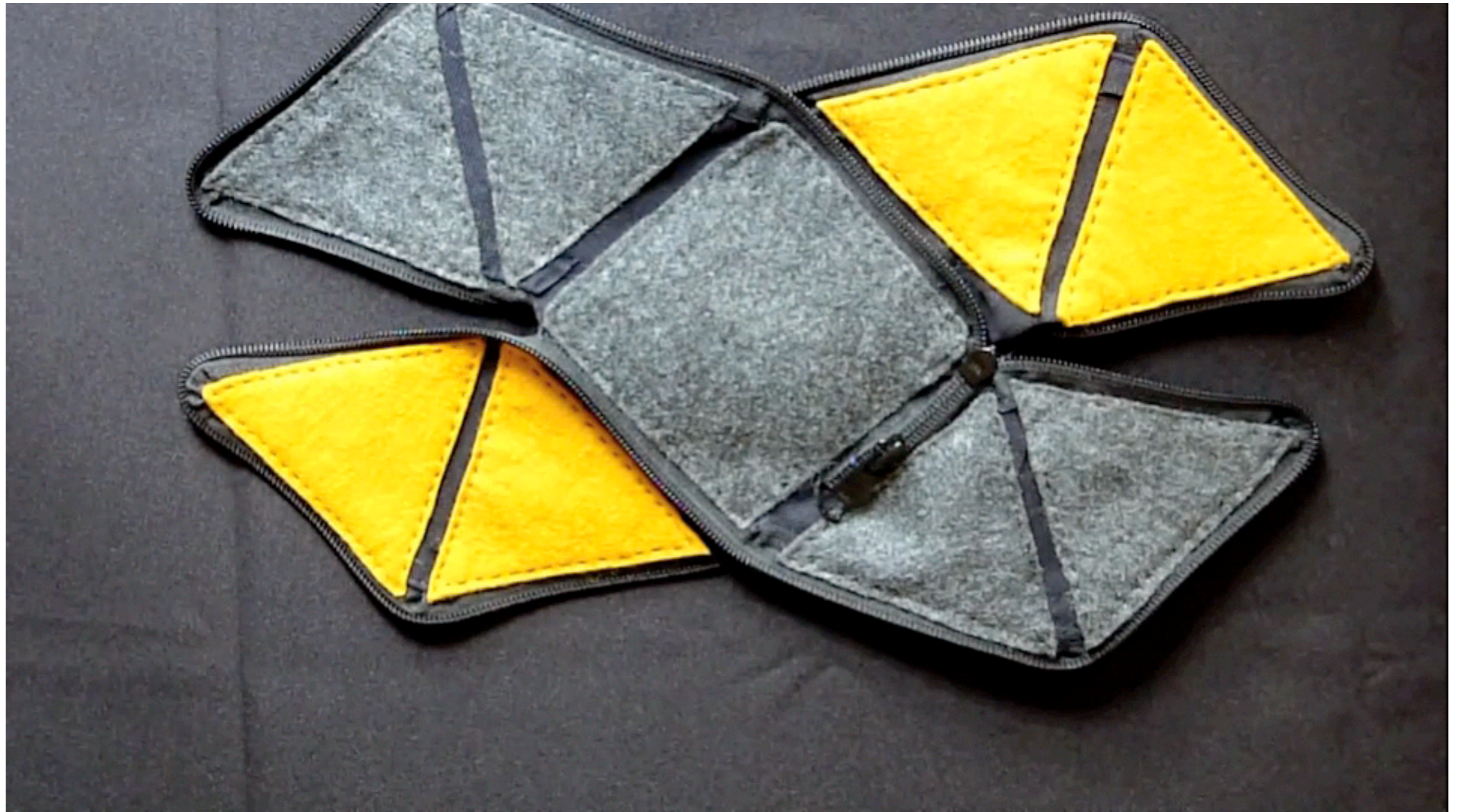
not allowed



on the polygon



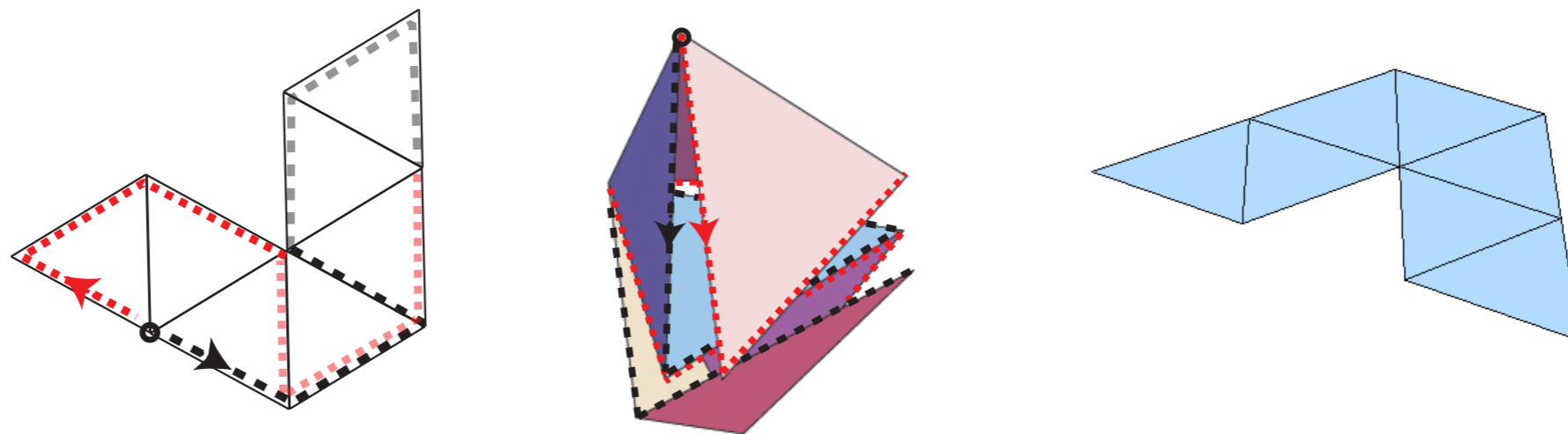
Polyhedral Complexes—Zipper Unfolding



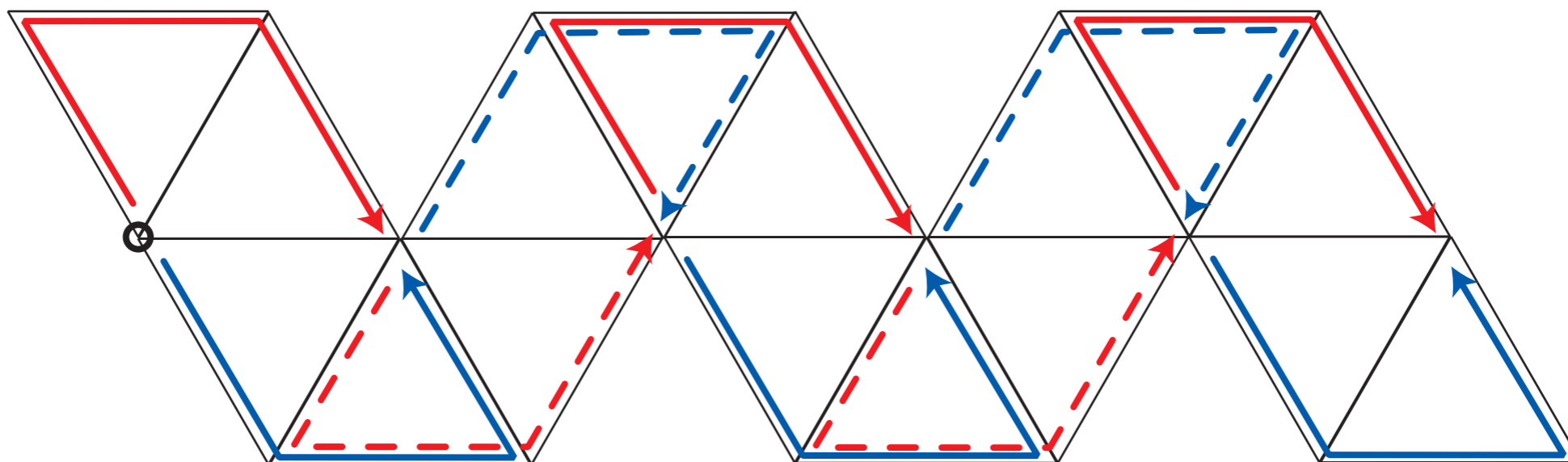
Polyhedral Complexes—Zipper Unfolding

Polyhedra Sharing Faces

2 tetrahedra



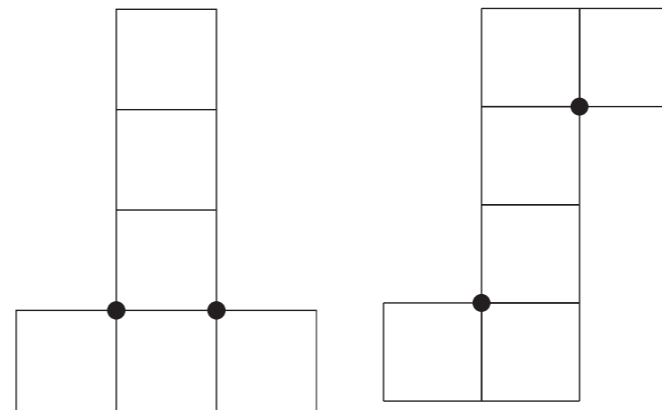
what will this zip into?



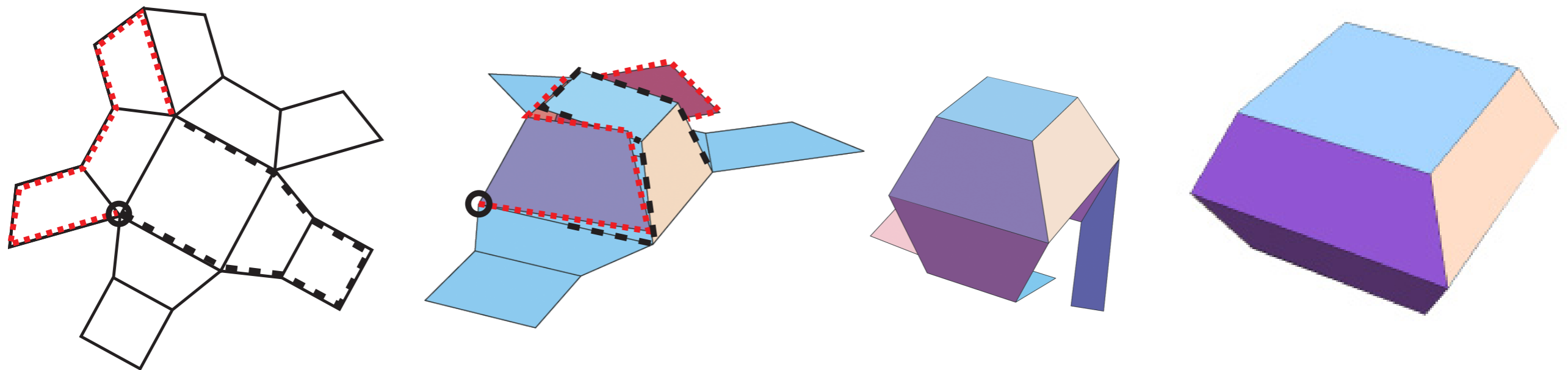
Polyhedral Complexes—Zipper Unfolding

Polyhedra Sharing Faces

2 cubes—seems impossible



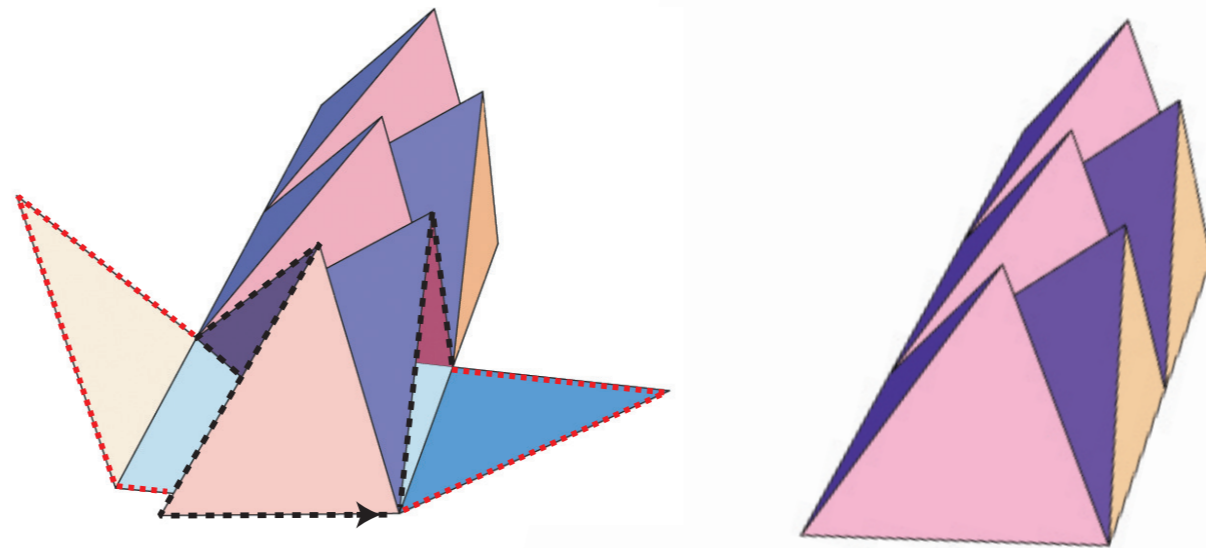
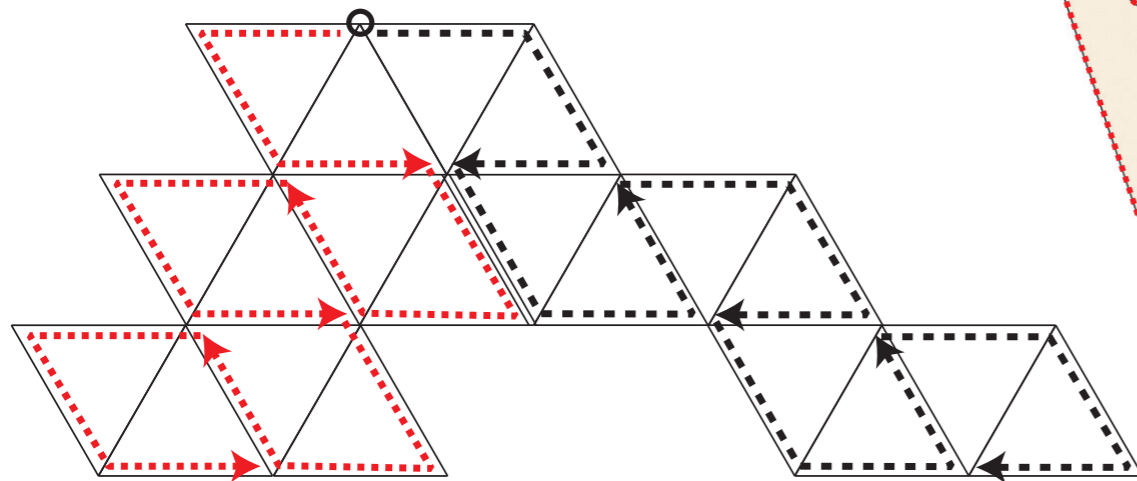
2 squashed cubes



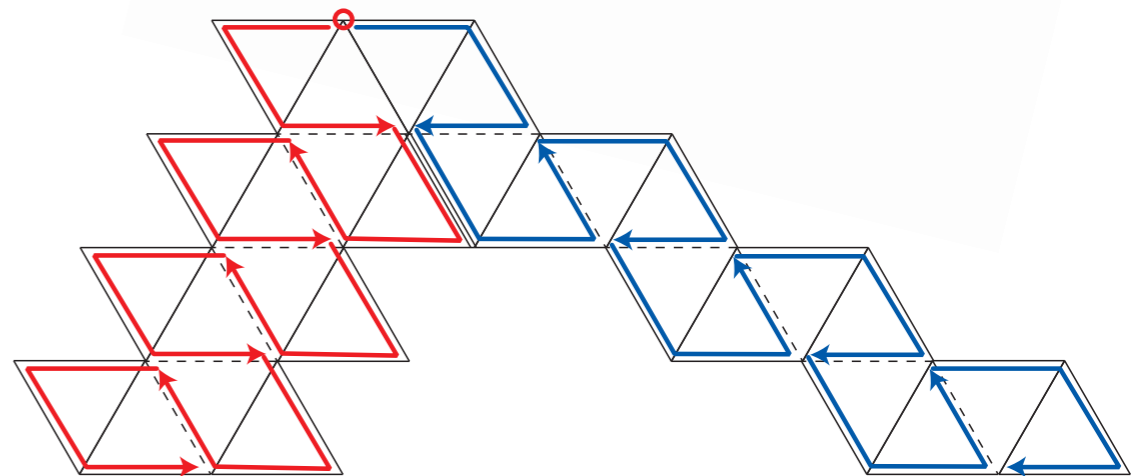
Polyhedral Complexes—Zipper Unfolding

Polyhedra Sharing Edges

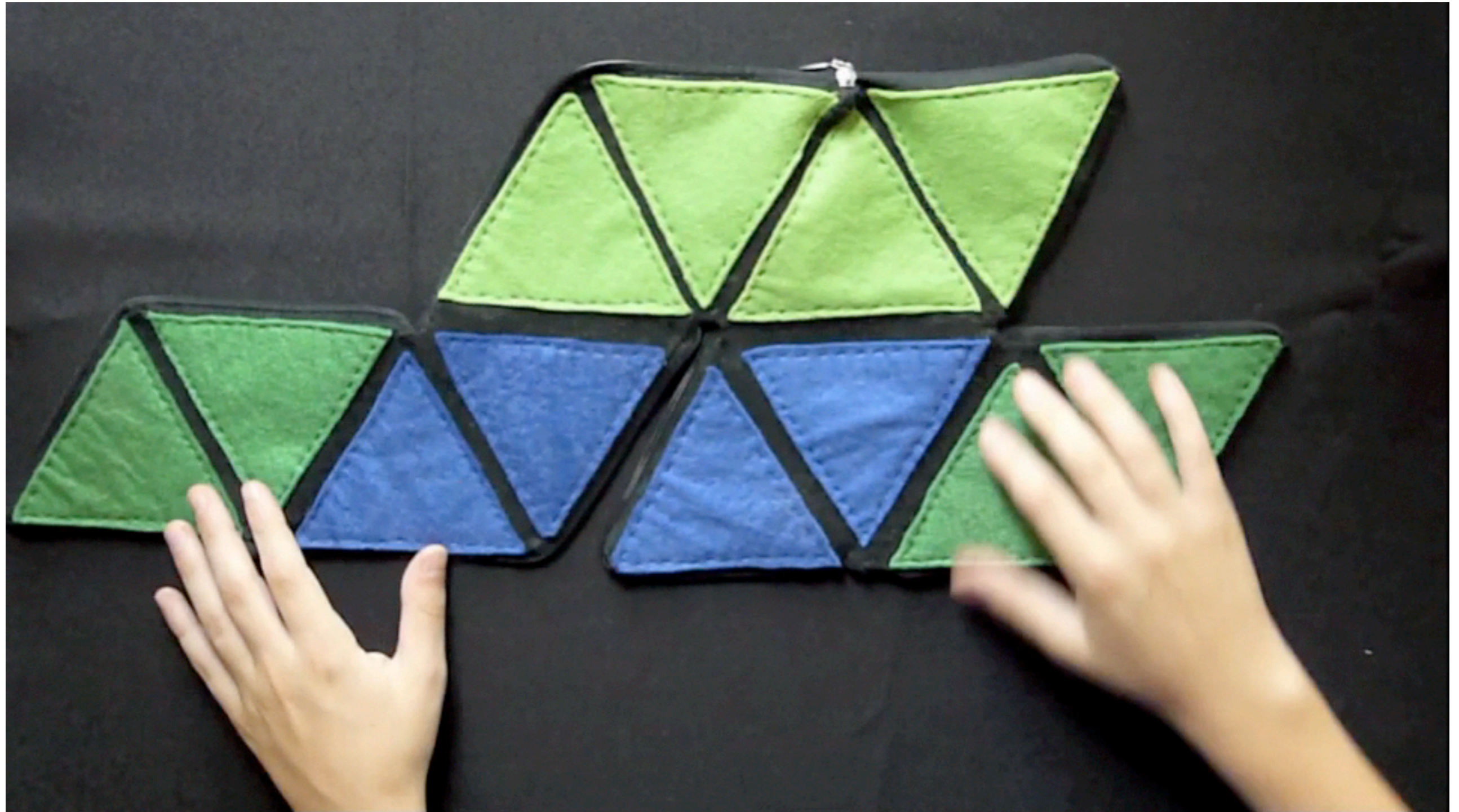
chain of tetrahedra sharing adjacent edges



can extend to n tetrahedra



Polyhedral Complexes—Zipper Unfolding



Open Problems

- Does every convex polyhedron have an edge unfolding?
- Does every polyhedron have a [general] unfolding? Every polyhedral manifold?

New:

- Does every convex polyhedron have a zipper unfolding?
- Show it's NP-hard to recognize torus polyhedra with zipper edge unfoldings.
- Which polyhedral complexes have [zipper/edge] unfoldings?

Zipper Unfoldings of Polyhedral Complexes

Erik Demaine Martin Demaine
Anna Lubiw Arlo Shallit Jonah Shallit